3rd Sem BTD NOTES - Rohini. H.

MODULE - 1

DATE:

PAGE:

Thermodynamics is the science of energy transfer relations b/w heat, work and the properties of the system.

why do we need these interactions.

coz all forms of energy are not the same.

- Shaft work.
- Electrical energy.
- comparing diff^n currencies.
- Heat / cool up room.
- Petrol in a car engine.
- Power plant to generate electricity

Matter ⇒ It has mass & takes up space (mass & volume

Thermodynamics ⇒ thermal (heat) Dynamic (force)

Laws of conservation of energy ⇒ energy can neither be created nor be destroyed.

Macroscopic approach ⇒ It is a quantity of matter considered without a concern about the events occurring at the molecular level. eg: pressure, temperature

Microscopic approach ⇒ It is the effect of molecular motion occurring in matter. eg: velocity, position & energy i.e the temperature of a gas is a function of the velocity of molecules. most of the microscopic properties cannot be measured with common instruments nor can be perceived by human senses

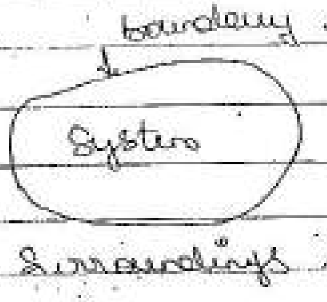
Property ⇒ It is one of the characteristics of the system to which some physically meaningful numbers are assigned.

eg: volume, temperature, pressure etc. These are all macroscopic in nature. When all the properties of a system have definite values, the system is said to exist at a definite state. Properties are the coordinates to describe the state of a system.

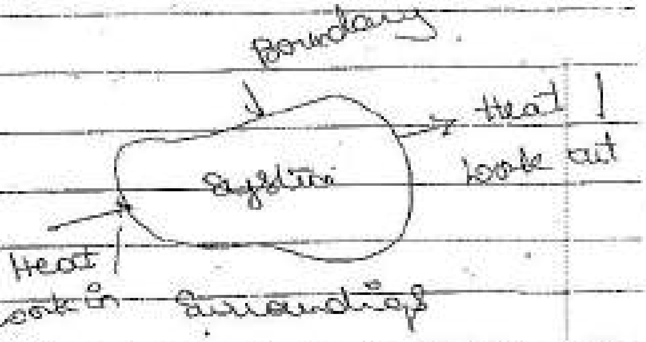
System :- To study thermodynamics, we choose a small part of the universe to which we will apply the laws of thermodynamics.
A thermodynamic system is defined as a quantity of matter or a region in space upon which study is carried out.
eg: the water kettle or aircraft engine.

Surroundings :- Everything external to the system is called the surroundings or the environment. Anything outside the system close enough to the system to have some effect on the system is called the surround

Boundary :- The surfaces which separates the system from the surroundings are called the boundaries. A system and its surroundings together comprise a universe. A boundary may be either fixed or moving.



A thermodynamic System

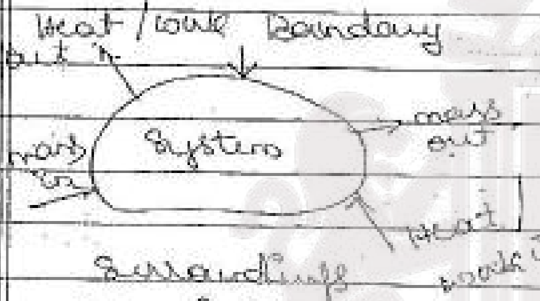


A closed System

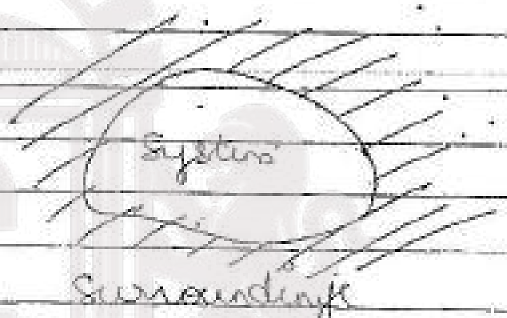
Types of System :-

(a) Closed System :- It is a system of fixed mass. There is no mass transfer across the system boundary i.e. heat transfer takes place but not mass.
eg: fluid in a piston constitute a closed system.

(b) open system :- It is a system where mass can cross the system boundary in either direction
eg: An air compressor in which air enters at low pressure & leaves at high pressure.



Open System



Isolated System

(c) Isolated System :- It is a system where there is no interaction b/w the system and the surroundings. It is of fixed mass and energy. i.e. there is no mass or energy transfer across the system boundary.

Properties

(1) Intensive properties :- They are independent of the mass in the system or size. eg: pressure, temperature.

(2) Extensive properties :- They are dependent on mass or size of the system eg: Volume, Mass (V, M)

(3) Specific property :- It is the value of an extensive property per unit mass of system. eg: Specific volume, density etc

State :- It is the condition of a system as defined by the values of all its properties. It gives a complete discription of the system.

Phase :- It is a quantity of mass that is homogeneous throughout in chemical composition and physical structure.
eg: solid, liquid & gas/vapour.

Path :- The locus of the series of states through which a system passes from initial state to its final state.

A thermodynamic process represents a transition in which a system changes from one state to another.

Types of processes.

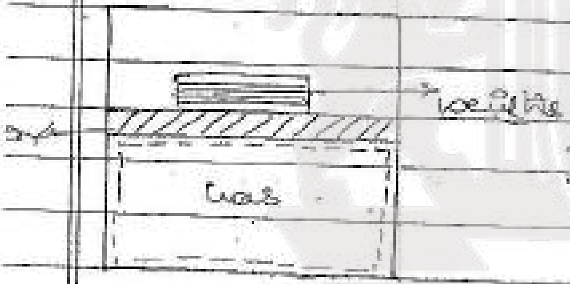
As a matter of rule we allow one of the properties to remain a constant during a process. we construct as many processes as by keeping different property as constant.

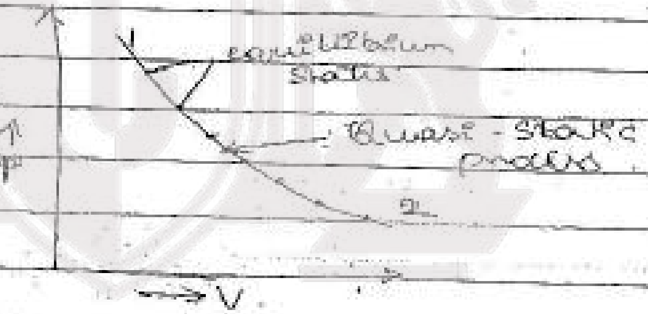
- Isothermal (T)
- Isochoric (V)
- Isenthalpic (h)
- Isobaric (P)
- Isentropic (S).
- Isosteric (conc)
- Adiabatic (no heat addition or removal).

Quasi-static processes.

Consider a system of gas contained in a cylinder fitted with a piston upon which very small pieces of weights are placed. The upward force exerted by the gas just balances the weights on the piston. The system is initially in equilibrium state identified by pressure P_1, volume V_1 & temperature T_1. When these weights are removed slowly, one at a time, the unbalanced potential is infinitesimally small. Every state passed by the system will be an equilibrium state. The locus of a series of such equilibrium states is called a quasi-static or quasi equilibrium process.

If all the weights are removed suddenly, the unbalanced potential would be finite. The piston will jump up, strike the stoppers & finally come to equilibrium state 2 at pressure P_2, volume V_2, temperature T_2 after many oscillations.





Equilibrium states   Quasi-static Process.

Cyclic and non-cyclic process.

When a system in a given state undergoes through a series of processes such that the final state is identical with the initial state, a cyclic process or a cycle.

If the system does not restored after the processes, then it is a non-cyclic process.

Equilibrium State :- If its properties will not change without some effect by the surroundings.
If all the properties in the system are uniform, then it is in equilibrium.

eg: Heat flows from a body at higher temperature to the one at a lower temperature.
Mass transfer occurs from higher concentration to a lower one.

Between the system and surroundings, if there is no difference in
Pressure → Mechanical eqn.
Potential → Electrical eqm.
Temperature → Thermal eqm.

Thermal Equilibrium.
If the temperature is same throughout the entire system and the system is isolated from its surroundings, the system is said to be in thermal equilibrium.

Thermodynamic Equilibrium
A system which is simultaneously in a state of mechanical equilibrium, thermal equilibrium, & chemical equilibrium is said to be in a state of thermodynamic equilibrium.

(a) Mechanical Equilibrium.
Condition (or) State in which there is no unbalanced force within the system and nor at its boundaries, eg the weight on the piston from the

piston-cylinder assembly is removed, the system boundary will expand until the mechanical equilibrium is attained. If there is no change in pressure at any point of the system with time and the system is isolated from its surroundings the system is said to be in mechanical equilibrium.

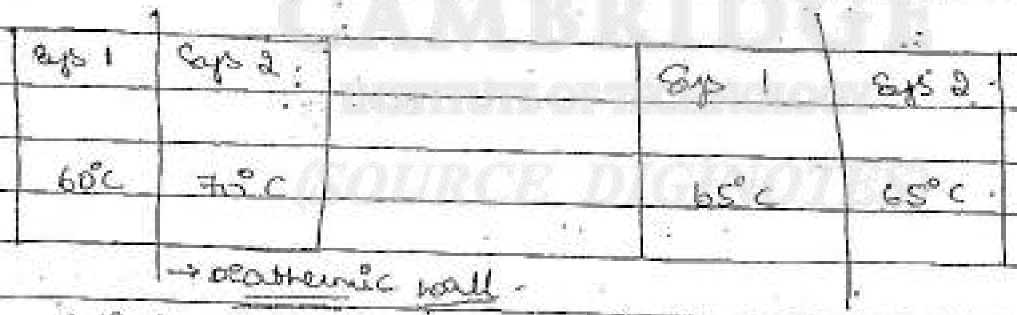
(b) chemical Equilibrium.

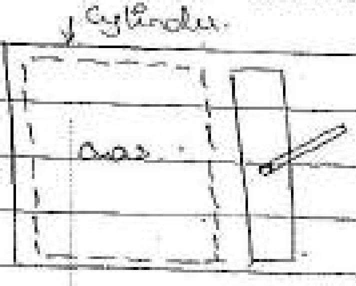
A system in mechanical equilibrium may undergo a spontaneous change of internal structure due to chemical reaction or diffusion.

If there is no change in chemical composition of the system with time i.e no chemical reaction or diffusion occurs, the system is said to be in chemical equilibrium. [no mass diffusion]

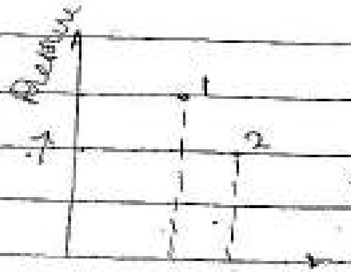
(c) Thermal equilibrium.

If two systems are separated by a diathermic wall, the temperature will change spontaneously until an equilibrium state of the combined system is attained. Two systems are then said to be in thermal equilibrium with each other.

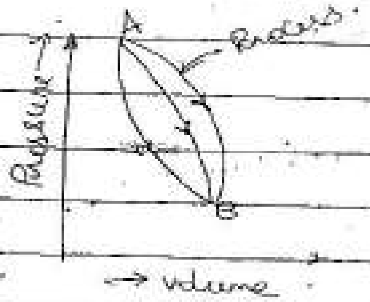




State



Process



System Path

Reversible and Irreversible Processes.

A thermodynamic process is reversible if the system passes through a continuous series of equilibrium state. An equilibrium state of the system can be located on any diagram drawn properties at any instant during the process.

An reversible process should satisfy the following conditions

- There should be no friction.
- The heat exchange to or from the system.
- The process should be quasi static

Salient features.

- It leaves no trace of its occurrence in the system and surroundings.
- It can proceed in either direction without violating the second law of thermodynamics.
- A reversible process yields the maximum work in engines and requires minimum work in devices such as refrigerators, pumps & compressors

Irreversible process.

A process is irreversible if the system passes through a sequence of non-equilibrium state. During such a process, the properties of the system donot have a unique value. When an irreversible process is made to proceed in the backward direction, the original state of the system is not restored.

An irreversible process is identified by the following characteristics.

- It can be carried out in one direction only.
- It occurs at a finite rate.
- It cannot be reversed without causing permanent changes in the surroundings
- the system is never in equilibrium state at any instant during an irreversible process.

Ex:

- Electric ckt with resistance.
- Spontaneous chemical reaction.

Zeroth law of Thermodynamics -

It states "If two bodies are in thermal equilibrium with the third body, they are also in thermal equilibrium with each other.

Temperature

It is a measure of 'hotness' or 'coldness' of a substance. [mean kinetic energy of the molecules of the subs]

Equality of Temperature

Two systems are equal in temperature if no change occurs in any property when they are brought together.

Let two blocks of copper, one hot and the other cold, be brought into thermal eq communication with each other. Each block is in contact with mercury in glass thermometer. After some time, ht of the mercury in both the thermometers is same, therefore two bodies have equality of temperature.

Consider two systems S_1 & S_3 which are in thermal equilibrium with each other. It is assumed that the systems S_1 & S_3 donot react each other chemically or electrically. Let the system S_3 be separadily equal in temperature to another system S_2 so that no change occurs when they communicate with each other. It is then observed that the systems S_1 and S_2 will not undergo any physical changes when brought into contact. The systems S_1 & S_2 are then taken to be in thermal equilibrium

Thermometry

It may be defined as the act of measuring temperature with accuracy and precision. A property or characteristics which changes in value as a function of temperature is called thermometric property and the corresponding substance is known as thermometric substance.

The physical characteristics which are so used could be

- a change in dimensions, i.e expansion, contraction of material in the form of the solid, liquid or gas.
- a change in electrical resistance of metals & semi conductors
- a thermo-electric emf for two different metals or alloys joined together.

A function of materials when exposed to temperature.

Temperature Scales

A quantitative measure of the temperature of a system requires reference to some datum plane or reference condition, and the establishment of a suitable temperature unit.

The important temperature scales are listed below:

Centigrade and Fahrenheit Scales

In both these scales the ice point & the steam point at one standard atmospheric pressure are used as fixed points.

Celsius Scale → The temp^r of ice and steam points are fixed at 0° & 100°C. There are 100 degrees b/w the ice point & the steam point.

Fahrenheit Scale → The temp^r of ice and steam points are 32°F & 212°F. There are 180 degrees b/w the ice point & the steam point.

Ice point → The equilibrium temperature of ice with air-saturated water at standard atmospheric pressure and is assigned a value of 0°C.

Steam point → The equilibrium temperature of pure water with its own vapour at standard atmospheric pressure and is assigned a value of 100°C

Kelvin Scale → A value of 273.15K, 373.15K & 273.16K are assigned to ice point, steam point and triple point.

Rankine Scale → A value of 491.67R, 671.67R & 491.69R are assigned to ice point, steam point and triple point of water

Dec 2013 | Jan 2014.

Define a quasi-static process. A platinum wire is used as a resistance thermometer. The wire resistance was found to be 10Ω & 16Ω at ice pt and steam pt respectively, and 30Ω at sulphur boiling pt of 444.6°C. Find the resistance of the wire at 750°C, if the resistance varies with temperature by the relation. $R = R_0(1+\alpha t+\beta t^2)$.

Soln:- $R = R_0(1+\alpha t+\beta t^2)$.

$10 = R_0(1+0+0)$ at ice point

$\therefore \boxed{R_0 = 10}$.

$16 = 10(1+\alpha 100+\beta 100^2)$.

$16 = 10+\alpha 100+\beta 100^2$

$6 = \alpha 100+\beta 100^2$ at steam point → (1)

$30 = 10(1+\alpha 444.6+\beta 444.6^2)$.

$30 = 10+\alpha 444.6+\beta 444.6^2$

$20 = \alpha 444.6+\beta 444.6^2$ at sulphur point → (2)

Solving (1) & (2)

$20 = \alpha 444.6+\beta 444.6^2$

$6 = \alpha 100+\beta 100^2$

$$\boxed{\begin{aligned}\alpha &= 6.436\times10^{-3}\\ \beta &= -4.357\times10^{-6}.\end{aligned}}$$

$R = 10[1+6.436\times10^{-3}\times750-4.356\times10^{-6}\times750^2]$

$\boxed{R = 33.76\,\Omega}$

June / July 2014.

Define a new temp^r scale N in which freezing point & boiling point of water are 100°N & 300°N respectively. Correlate this temp^r scale with centigrade scale for which freezing and boiling points are 0°C & 100°C respectively.

$100°N = at + b$. at ice pt.

$100°N = a(0) + b$.

$\boxed{100 = b}$

$300°N = at + b$.

$300 = a(100) + b$.

$300 = a100 + 100$.

$\therefore \boxed{a = 2}$.

$\therefore \boxed{N = 2T + 100}$

where T is in °C.

Jan 2016.

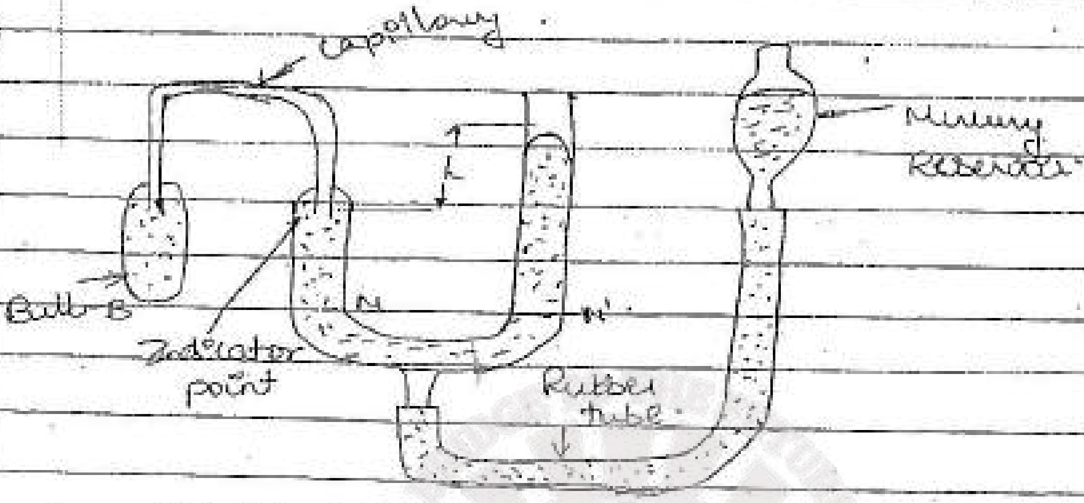
A constant volume gas thermometer containing helium gas gives readings of gas pressure 1000mm and 1366 mm of mercury at the ice point and steam point respectively. (i) Express the gas thermometer Celsius temp^r in terms of gas pressure.

(ii) The thermometer when left standing in the atmosphere, registers 1075mm of mercury.
Determine the atmospheric temp^r.

In 1701 Newton proposed a linear temperature scale in which the ice point temp^r was taken as 0°N & the human body temp^r was taken as 12°N. Find the conversion scale b/w Newton's scale of temp^r & Centigrade scale of temp^r if the temp^r of human body in centigrade scale is 37°C.

A constant volume gas thermometer containing H_2 records a gas pressure P as 1200 & 1400 mm of Hg at ice and steam point respectively. Assume a linear relationship $t = ap + b$. Derive (i) a relation for gas thermometer celsius temp^r 't' in terms of 'P'. If pressure recorded by the gas thermometer is 1050mm Hg.
What is the temp^r in °C.

The resistance of a platinum wire is found to be 11 Ω at the ice point, 15.24 Ω at the steam pt & 28.887 Ω at the sulphur point. Find (i) the constants A & B in the equation $R_0[1+Bt+Ct^2]$.



Constant volume gas thermometer.

A gas is contained in the bulb B (made of platinum or its alloy) which communicates with the mercury column M through a capillary. The volume of the gas is kept constant by adjusting the ht of the mercury column M until the mercury level just touches the tip of a small pointer in the space above M known as the dead space. The mercury column M is adjusted by raising or lowering the reservoir. The difference in height b/w the two mercury columns M & M' is measured when the bulb is surrounded by tempr t and when it is surrounded by the water at the triple point. The pr of the gas is equal to atmospheric pressure + h. Tempr is given by $\boxed{t = 273.16\,(P/P_t)}$.

Constant pressure gas thermometer.

In this thermometer the mercury has to be adjusted to keep pressure constant and the volume of gas V which would vary with the temp of the system and becomes the thermometric property. Then the reqd temp can be calculated using the eqn

$$\boxed{T = 273.16\,\frac{V}{V_{tp}}}$$

Problems

1. The temperature t on a thermometric scale is defined in terms of a property P by the relation,
$t = a \log_e P + b$. where a and b are constants.
The temperatures of the ice point and steam point are assigned the numbers 32 and 212 respectively. Experiment gives values of P of 1.86 and 6.81 at the ice point & steam point respectively. Evaluate the temperature corresponding to a reading of 2.50 on the thermometer.

Soln: $32 = a \log_e 1.86 + b$
$212 = a \log_e 6.81 + b$
By solving ∴ $a = 138.7$ & $b = -54.07$
$t = 138.7 \log_e P - 54.07$
when $P = 2.50$
$t = 138.7 \log_e 2.50 - 54.07 = \underline{73^\circ}$

June 2012 (10)

2. The readings t_A & t_B of two thermometers A and B on the celsius scale agree at the ice point and the steam point, and are assumed to be related by the expression $t_A = l + m t_B + n t_B^2$, where l, m and n are constants. When both thermometers are immersed in a well stirred oil bath, A reads 51°C & B reads 50°C. Determine the reading on thermometer A when thermometer B reads 25°C. Which thermometer can be taken to be correct.

Soln: At ice point: $t_A = t_B = 0^\circ C$
∴ $0 = l + m \times 0 + n \times 0^2$; $l = 0$.
At steam point: $t_A = t_B = 100^\circ C$
∴ $100 = 0 + m \times 100 + n \times (100)^2$
$100 = 100m + (100)^2 n$
(or) $m + 100n = 1 \longrightarrow (1)$

Also when $t_A = 51°C$, $t_B = 50°C$, we have.

$$51 = 0 + m \times 50 + n \times (50)^2$$
$$= 50m + (50)^2 n$$
$$m + n50 = \frac{51}{50} \longrightarrow (2)$$

Solving eqns (1) & (2)

$$m = 1.04 \text{ and } n = -0.0004.$$

when $t_B = 25°C$, we have.

$$t_A = 1.04 \times 25 - 0.0004 \times 25^2$$
$$t_B = 25.75°C \quad (\text{from above}).$$

(i) one of the thermometer may be correct, (or)
(ii) Both are faulty ...

Dec 2011 (08)

3) The emf thermocouple with the test junction at $t°C$ on gas thermometer scale and reference junction at ice point is given by

$$e = 0.20t - 5 \times 10^{-4} t^2 \, mV.$$

The millivoltmeter is calibrated at ice and steam points. what will the thermometer read in a place where the gas thermometer reads 50°C

Soln: At ice point, when $t = 0°C$, $e = 0mV$

At steam point, when $t = 100°C$

$$e = 0.20 \times 100 - 5 \times 10^{-4} (100)^2$$
$$= 15mV.$$

At $t = 50°C$: $e = 0.20 \times 50 - 5 \times 10^{-4} (50)^2 = 8.75mV$

when the gas thermometer reads 50°C, the thermocouple will read: $\frac{100}{15} \times 8.75$, (or) $\underline{58.33°C}$ //

$$t°C = 100 \left[\frac{e - e_I}{e_s - e_I} \right] \qquad t°C = 100 \left[\frac{8.75 - 0}{0 - 15} \right]$$

4) The length h of the mercury column of a mercury in glass thermometer records 5.0cm (at ice point) and 20.0cm Hg (at steam point). The temperature scale ($t°C$) and property (length of the mercury column h) are related by

$$t = ah^2 + b$$

where $t = 0$ at ice point, $t = 100°C$ at steam point. Compare this scale with Celsius scale where t is linearly defined in terms of h with the same values of t at the ice and steam point.

Soln: At ice point, $t=0$ for given relation $t = ah^2+b$

$$0 = a5^2 + b \quad \therefore b = -25a$$

At steam point, $t = 100$

$$100 = a20^2 + b \quad \therefore b = -400a + 100$$

Solving we get $a = 0.267$, $b = -6.675$

Substituting values of a & b

$$t = 0.267h^2 + 6.675$$

Let the Celsius scale is given by relation

$$t^* = Ph + Q$$

At ice point, $0 = P \times 5 + Q \quad \therefore Q = -5P$

At steam point, $100 = P \times 20 - 5P \quad \therefore Q = -33.33$

$$\therefore t = 6.667h - 33.33$$

$$\therefore \frac{t}{t^*} = \frac{0.267h^2 - 6.675}{0.667h - 33.33}$$

5) Temperature Scales.

Consider a thermometric property X such that the temperature t - is a linear function of this property.

$$t = a + bX.$$

where a and b are constants, which can be obtained by measuring X at two known temperatures.

celsius (centigrade scale).

ice-point $0 = a + bX_I$

$$100 = a + bX_S$$

$$100 = b(X_S - X_I)$$

~~$b = X_S - X_I$~~ $b = \frac{100}{X_S - X_I}$ ...... (2)

$a = -bX_I \Rightarrow a = -\left[\frac{100X_I}{X_S - X_I}\right]$ ...... (3)

$$\therefore t^\circ C = \frac{-100X_I}{X_S - X_I} + \frac{100X}{X_S - X_I}$$

$t^\circ C = 100\left[\frac{X - X_I}{X_S - X_I}\right]$ ...... (4).

Fahrenheit Scale

$$32 = a + bX_I.$$

$$212 = a + bX_S$$

$$180 = b(X_I - X_S)$$

$b = \frac{180}{[X_I - X_S]}$ ...... (5).

$a = -bX_I = 32 - \frac{180X_I}{X_I - X_S}$ .... (6)

$$\therefore t^\circ C - 32 = \frac{180X_I}{X_I - X_S} + \frac{180X}{X_I - X_S}$$

$t^\circ F = 32 + \left[\frac{X - X_I}{X_S - X_I}\right] \times 180$ .... (7)

Relationship between t°C & t°F.

$$\frac{X-X_i}{X_s-X_i} = \frac{t°C}{100}$$ from eqn (4) .... (8)

from (7) & (8).

$$t°F = 32 + \left[\frac{t°C}{100}\right]180.$$

$$t°F = 32 + \frac{9}{5}t°C$$

$$t°C = (t°F - 32)\frac{5}{9}.$$

||ly we have

$$t(K) = t°C + 273.15$$

$$t(R) = t(°F) + 459.67$$

$$t(R) = 1.8t(K)$$

$$t(°F) = 1.8t(°C) + 32$$

Thermometric property → A physical property that changes in a known way with temperature, & can therefore be used to measure temperature.

Thermometer	Thermometric property
Gas (constant volume.	Pressure
Electric resistance thermometer	Electric resistance.
Thermocouple	Thermal emf.
Radiation thermometer	Radiant emittance.
Liquid column in glass capillary	length

Devices used for measurement of temperature.

- * constant volume gas thermometer.
- * Electric Resistance thermometer.

July 2013 (10)

12) Develop a linear temperature scale °B where ice and normal human body temperature are assumed as two fixed points and assigned the values 0°B & 50°B respectively. If the temperature of human body on celsius scale is 36.7°C, obtain the relation b/w 'B' scale and celsius scale and find out water boiling temperature in 'B' scale.

$B^{\circ} = At + C.$

at $B = 0,\ t = 0$ $\quad 0 = A(0) + C$

$\therefore \boxed{C = 0}$

At $B = 50,\ t = 36.7$

$\therefore 50 = 36.7(A) + 0.$

$\boxed{A = 1.362}$

At boiling point $t = 100°C$

$°B = 1.362 \times 100 = 136.2°B$

$\boxed{°B = 136.2}$

06 - June 2010

13) A thermometer is calibrated with ice and steam points are fixed points referred to as 0°C and 100°C respectively. The equation used to establish the scale is $t = a \log_e x + b$

i) Determine the constants 'a' & 'b' in terms of ice point (x_i) and steam point (x_s).

ii) S.T $t°C = 100 = \dfrac{\log_e\left(\frac{x}{x_i}\right)}{\log_e\left(\frac{x_s}{x_i}\right)}$

$t = a \log_e x + b.$

ice pt $\quad 0 = a \log_e x_i + b.$

$100 = a \log_e x_s + b$

Solution $100 = a(\log_e x_s - \log_e x_i)$.

$a = \dfrac{100}{\log_e\left(\frac{x_s}{x_i}\right)}$. $\therefore b = -a \log_e x_i$.

$b = -\dfrac{100 \log_e x_i}{\log \frac{x_s}{x_i}}$

Substituting

$t = \dfrac{100 \log x}{\log_e \frac{x_s}{x_i}} - \dfrac{100 \log_e x_i}{\log_e \frac{x_s}{x_i}}$

Given $= 100 \left[\dfrac{\log_e \frac{x}{x_i}}{\log_e \frac{x_s}{x_i}}\right]$ q

July 2011 (06)

5) A temperature t on a celcius scale is defined in terms of property p by the relation $P = e^{(t-B)/A}$, where A & B are constants. Experiments gives value of P of 1.86 and 6.81 at the ice & steam point respectively. obtain relation for t and also find the temp t for the reading of P = 2.5.

At Ice pt $1.86 = e^{-B/A}$; $\ln 1.86 = -B/A$.

Steam pt $6.81 = e^{\left(\frac{100-B}{A}\right)}$; $\ln 6.81 = 100 - B/A$

$0.3235 = \dfrac{B}{-100+B}$.

$\therefore B = \dfrac{-32.35}{0.6765} = \underline{-47.82}$; $e^{\frac{47.82}{A}} = 1.86$.

$\dfrac{47.82}{A} = \ln 1.86$

$\boxed{\therefore A = 77.12}$

For reading at P = 2.5

$2.5 = e^{\frac{t+47.82}{77.12}}$.

$0.9163 = \dfrac{t+47.82}{77.12}$ $\boxed{\therefore t = 22.84°C}$

Rohini A **WORK AND HEAT** Ist module [illegible]

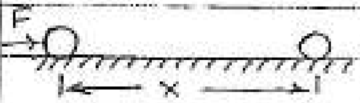
BTP notes DATE: PAGE: 30

[illegible] mechanical

WORK

work is done when the point of application of a force moves in the direction of force. The amount of work done equals the product of force and the resultant displacement in the direction of force.

fig(i)



fig(ii)

F is the force acting on a body and the body moves a distance 'x' in the direction of force. Then the work done on the body is given by

work done = force × distance ⇒ $W = Fx$ (fig(i))

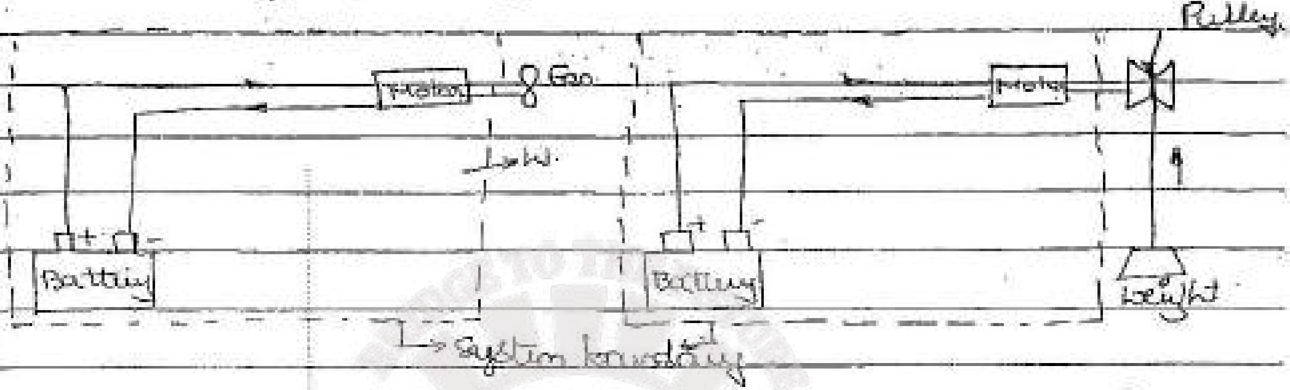
W.r.to fig(ii) consider the force F to be inclined at an angle θ with the direction of motion.

Then workdone = $F\cos\theta \times x$

Work :- It is one of the basic modes of energy transfer. Work refers to the interaction between system and surroundings whose sole effect, external to the system, could be reduced to the lifting of mass against the gravitational force. Work is the energy transfer by reasons other than temperature difference.

Let us consider the battery and the motor as a system. The motor is driving a fan. The system is doing work upon the surroundings. When the fan is replaced by a pulley and a weight, the weight may be raised with the pulley driven by the motor.

The sole effect on things external to the system is then the raising of a weight.



When work is done by a system, it is taken as positive and when work is done on a system, it is taken as negative. The unit of work is N.m or Joule. The rate at which work is done by, or upon the system is known as power. The unit of power is J/sec or watt.

(a) W is positive (b) W is negative

P.dV (or Displacement work

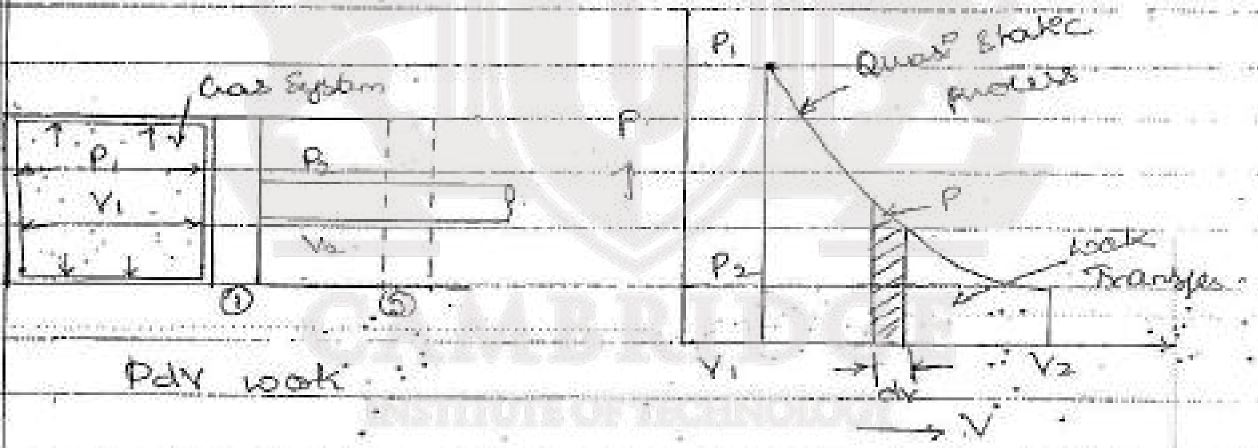
Consider a gas in the cylinder be a system having the pressure P_1 and volume V_1. The system is in thermodynamic equilibrium. The piston is the only boundary which move due to gas pressure. The state of gas in cylinder is P_1 & V_1 and the new final position 2 which is also in thermodynamic equilibrium which is specified by pressure P_2 and volume V_2

Let the pressure be P and the volume V at any intermediate point in the travel of the piston when the piston moves an infinitesimal distance dL and if 'a' be the area of the piston, the force F acting on the piston F = p.a. i.e. work done by the gas on piston is given by

$$dW = F\,dL = pa\,dL = p\,dV$$

where $dV = a\,dL$ = infinitesimal displacement volume. when the piston moves out from position 1 to position 2 with the volume changing from V_1 to V_2, the amount of work W done by the system will be.

$$W_{1-2} = \int_{V_1}^{V_2} p\,dV$$

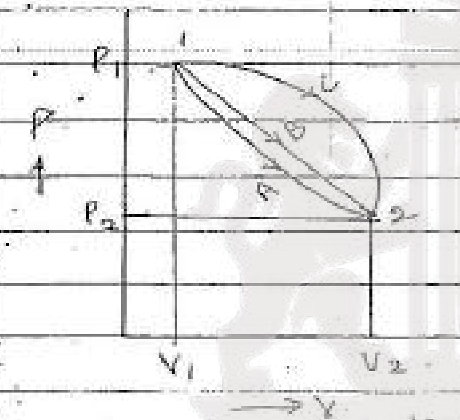


PdV work

The magnitude of the work done is given by the area under the path 1-2. Since p is a thermodynamic coordinate, the volume changes from V_1 to V_2 must be equilibrium states, i.e. the path 1-2 must be quasi-static. The piston moves infinitely slowly so that every state passed through is an equilibrium state. The integral $\int p\,dv$ can be performed only on a quasi static path.

Path function and Point function

From fig. It is possible to take a system from state 1 to state 2 along many quasi static paths. Such as A, B or C. Since the area under each curve represents the work for each process, the amount of work involved in each case is not a function of the end states of the process. And it depends on the path the system follows in going from state 1 to state 2. So we define work as a path function. i.e. dW is an inexact or imperfect differential



Work a path function

Thermodynamic properties are point functions, since for a given state, there is a definite value for each property. The change in a thermodynamic property of a system in a change of state is independent of the path the system follows during the change of state and depends only on the initial and final states of the system. The differentials of point functions are exact or perfect differentials & the integration is $\int_{V_1}^{V_2} dV = V_2 - V_2$

The change in volume depends only on the end states of the system irrespective of the path the system follows.

whereas, work done in a quasi-static process b/w two given states depends on the path followed.

$$\int_1^2 dW \neq W_2 - W_1$$

$$\int_1^2 dW = W_{2-1} \text{ or } W_{1-2}$$

To distinguish an inexact differential dW from an exact differential dV or dP the differential sign is being cut by a line at its top.

$$dV = \frac{1}{P} dW$$

Here, 1/P is called the integrating factor.

i.e when an inexact differential dW when multiplied by an integrating factor 1/P becomes an exact differential dV.

For a cyclic process the initial and final states of the system are the same, hence change in any property is zero.

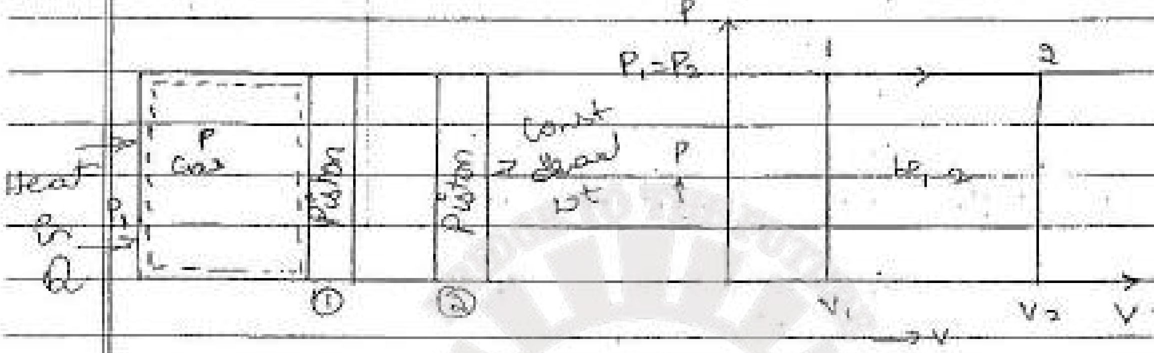
$$\oint dV = 0 , \oint dP = 0 , \oint dT = 0$$

Salient aspects of work.

1) The process must be quasi-static, reversible frictionless
2) The boundary of the system should move
3) There must be resisting force at the boundary.
4) The effects due to gravity, electric current & magnetisation are negligible

pdV - work in various Quasi-Static processes

1) Constant pressure process [Isobaric or isopiestic process]



$$W_{1-2} = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$$ W.D = Area under curve 1-2

2) Constant volume process [Isochoric process].

$$W_{1-2} = \int_1^2 P\,dV = P(V_2 - V_1) = 0 \quad [\because V_1 = V_2]$$

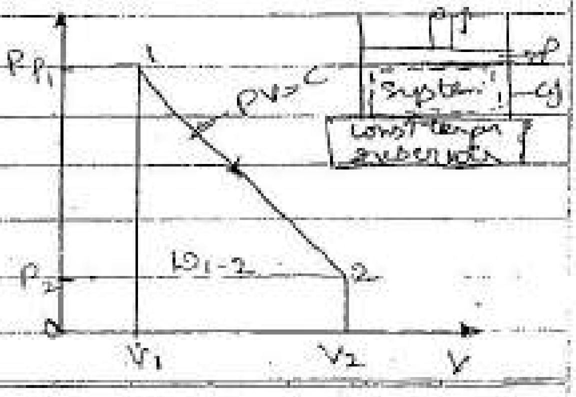
3) Constant Temperature process [Isothermal process].

$PV = C = P_1V_1 = C$

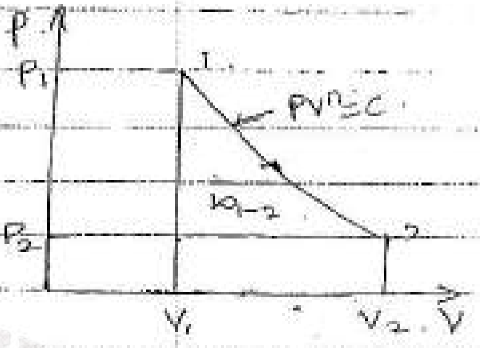
$P = \frac{P_1V_1}{V}$; $\frac{V_2}{V_1} = \frac{P_1}{P_2}$

$$W_{1-2} = \int P\,dV = \int_1^2 \frac{P_1V_1}{V}\,dV$$

$$= P_1V_1 \ln\frac{V_2}{V_1} = P_1V_1 \ln\frac{P_1}{P_2}$$



4) Polytropic Process:



$P_0V_0^n = P_1V_1^n = P_2V_2^n = C$

$P = \frac{(P_1V_1^n)}{V^n}$

$\therefore W_{1-2} = \int_{V_1}^{V_2} P\,dV$

$= \int_{V_1}^{V_2} \frac{P_1V_1^n}{V^n}\,dV$

$= (P_1V_1^n)\left[\frac{V^{-n+1}}{-n+1}\right]_{V_1}^{V_2}$

$= \frac{P_1V_1^n}{1-n}\left(V_2^{1-n} - V_1^{1-n}\right)$

$= \frac{P_2V_2^n \times V_2^{1-n} - P_1V_1V_1^{1-n}}{1-n}$

$= \frac{P_1V_1 - P_2V_2}{n-1} = \frac{P_1V_1}{n-1}\left[1 - \left(\frac{P_2}{P_1}\right)^{n-1/n}\right]$

$W_{1-2} = \int_1^2 \frac{C}{V^n}\,dV = \int_1^2 C\,V^{-n}\,dV$

$W_{1-2} = C\left[\frac{V^{-n+1}}{-n+1}\right]_1^2$

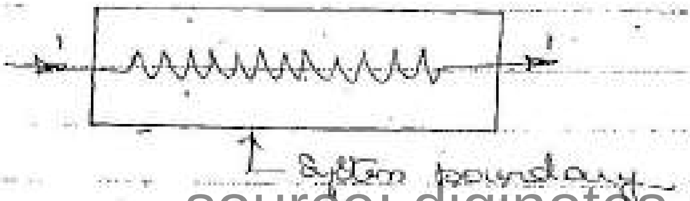
$W_{1-2} = C\left[\frac{V_2^{1-n}}{1-n} - \frac{V_1^{1-n}}{1-n}\right]$

$W_{1-2} = \frac{C}{1-n}\left[V_2^{1-n} - V_1^{1-n}\right]$

Other types of work Transfer

Electrical work:

When a current flows through a resistor taken as a system there is work transfer into the system. This is because the current can drive a motor, the motor can drive a pulley and the pulley can raise a load.



The current flow, I in amperes, is given by

$$I = \frac{dC}{d\tau}$$

where C is the charge in coulombs & τ is time in seconds. Thus dC is the charge crossing a boundary during time dτ. If E is the voltage potential, the work is

$$dW = E\,dC$$
$$= EI\,d\tau$$
$$\therefore W = \int_1^2 EI\,d\tau$$

∴ Electrical power will be

$$W = \lim_{d\tau \to 0} \frac{dW}{d\tau} = EI$$

This is the rate at which work is transferred.

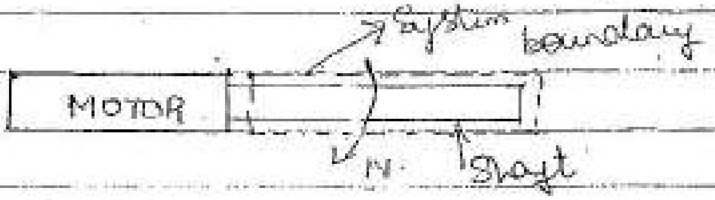
Shaft work :

When a shaft, taken as the system is rotated by a motor, there is work transfer into the system. This is because the shaft can rotate a pulley which can raise a weight. If T is the torque applied to the shaft & dθ is the angular displacement of the shaft, the shaft work is

$$W = \int_1^2 T\,d\theta$$

$$W = \int_1^2 T\frac{d\theta}{d\tau} = T\omega \ \text{kW (Power)}$$

where ω is the angular velocity & T is constant.



3) Paddle wheel (or Stirring work

As the weight is lowered, the paddle wheel turns, thereby work is transfer into the fluid system which gets stirred. Since the volume of the system remains constant $\int PdV = 0$

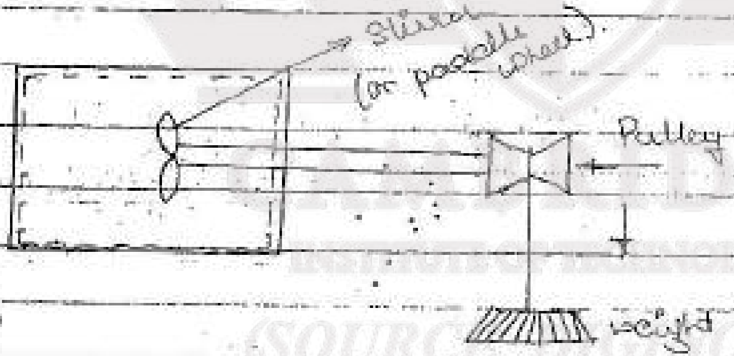
If m is the mass of the weight lowered through a distance dz and T is the torque transmitted by the shaft in rotating through an angle dθ, the differential work transfer to the fluid is given by,

$$dW = mg\,dz = T\,d\theta$$

and the total work transfer is

$$W = \int_1^2 mg\,dz = \int_1^2 W'dz = \int_1^2 T\,d\theta$$

where W' is the weight lowered.



4) Stretching of a wire

When a wire of C/S area A is subjected to an external force F, its length changes from l to (l+dl) and the work done is given by

$$dW = -F\,dl$$

The -ve sign indicate work done on the system to bring a change in its length

From mechanics of Solids,

Stress induced in the wire $\sigma = \frac{F}{A}$

Strain: $dl = \frac{dl}{l}$

Then $dW = -(\sigma A) \times l\,de$

and $W = -Al \int \sigma\,de$.

If extension of the wire is limited to within the elastic limits, we have

Modulus of elasticity $= \frac{\text{Stress}}{\text{Strain}}$; $E = \frac{\sigma}{e}$

$\therefore W = -AlE \int e\,de$

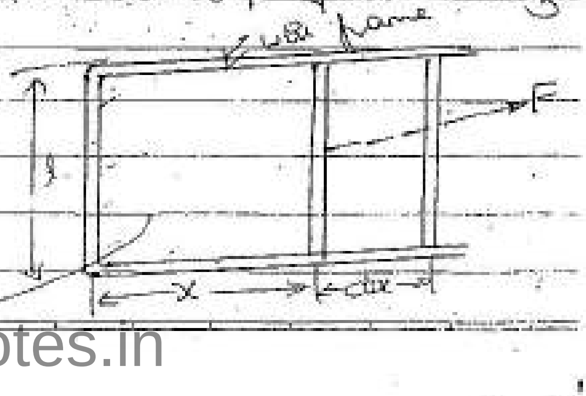
$W = -\frac{1}{2} AEl (e_2^2 - e_1^2)$

This equation is valid when the system is perfectly homogeneous and the extension (or compression) process is reversible.

5) Surface film

A film on the surface of a liquid is acted upon by the force of surface tension which tends to make the surface area of the liquid minimum. According to this the free liquid surface behaves like a stretched membrane. One of the liquid film held b/w a wire frame is moved with the help of a sliding wire.

$\therefore$ Force applied on the liquid film $F = 2l\sigma$



work done is given by,

$$W = -\int_{x_1}^{x_2} F dx = -\int_{x_1}^{x_2} 2l\sigma dx$$

The factor 2l dx represents a change in the surface area dA of the liquid film. Then

$$W = -\int_{x_1}^{x_2} 2l\sigma dx$$

$$= -\int_{A_1}^{A_2} \sigma dA = -\sigma(A_2 - A_1).$$

c) Magnetization of a paramagnetic solid.

The work done per unit volume on a magnetic material through which the magnetic and magnetisation fields are uniform is

$$\delta W = -H dI$$

and $W_{1-2} = \int_{I_1}^{I_2} H dI$

where H is the field strength, and I is the component of the magnetization field in the direction of the field. The negative sign provides that an increase in magnetization produces negative work.

Net work done by a system

It is the algebraic sum of all the processes executed by a system.

$\therefore W_{total} = W_{displ} + W_{stirring} + W_{electrical} + \ldots$

Heat

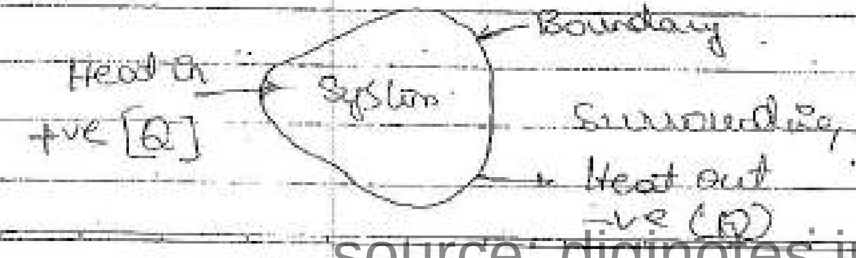
Heat is a form of energy that is transferred across the boundaries of a system because of temperature difference b/w the system and its surroundings i.e. always from a higher temperature to a lower tempr.

Heat Transfer

The transfer of heat b/w two bodies in direct contact is called conduction. Heat may be transferred b/w two bodies separated by empty space or gases by the mechanism of radiation through electromagnetic waves. A third method of heat transfer is convection which refers to the transfer of heat between a wall and a fluid system in motion.

The direction of heat transfer is taken from the high temperature system to the low temperature system. Heat flow into a system is taken to be positive and heat flow out of a system is taken as negative. The symbol Q is used for heat transfer.

Heat is a boundary phenomenon, since it occurs only at the boundary of a system. When heat is transferred to ice and water mixture, the temperature does not rise until all the ice has melted. Since a temperature rise may be caused by work transfer also. A process in which no heat crosses the boundary of the system is called an adiabatic process.



Heat Transfer - a Path function

The amount of heat transferred when a system changes from a State 1 to a State 2 depends on the intermediate states through which the system passes i.e its path. Therefore Q is an inexact differential,

$$\int_1^2 dQ = Q_{1-2} \text{ or } Q_{2-1}$$

$$W_{1-2} = \int_1^2 dW = \int_1^2 p\,dv$$

It is valid for a quasi static process and the work transfer is given under the area of path on P-V diagram.

The pressure difference is the cause & work transfer is the effect

Whenever there is a difference in temperature there will be heat flow. The temp^r difference is the cause and heat transfer is the effect.

$$Q_{1-2} = \int_1^2 dQ = \int_2^1 T\,dx$$

The amount of heat transferred when a system changes from a State 1 to a State 2 depends on the path the system follows.

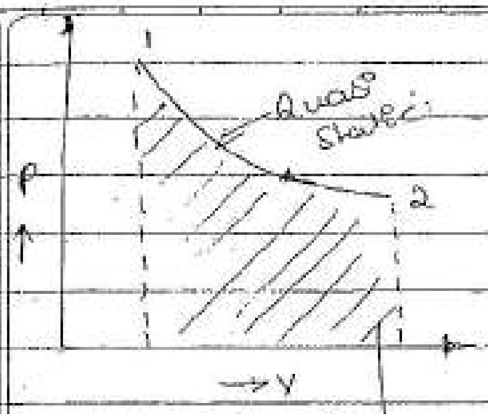
∴ dQ is an inexact differential

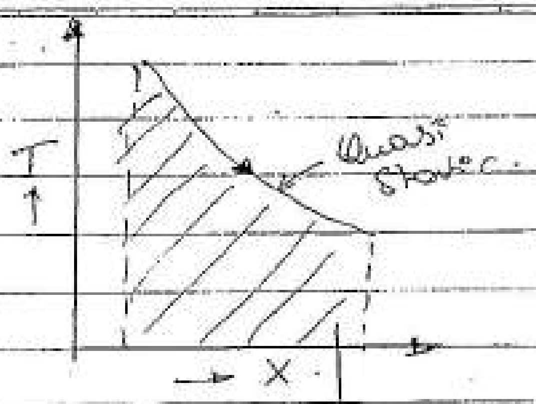
$$đQ = T\,dX$$

where X is an extensive property & dx is an exact diff^n

$$dX = \frac{1}{T}\,đQ$$

To make $đQ$ an exact differential, we introduced $1/T$ Integrating factor





$W_{1-2}=\int_1^2 PdV$ $\qquad$ $Q_{1-2}=\int_1^2 Tdx$

Area of work transfer. $\qquad$ Area of heat transfer.

Heat transfer is therefore a path function. i.e the amount of heat transferred when a system changes from a state 1 to a state 2 depends on the path the system follows.

The product of mass and specific heat (mc) is called the heat capacity of the substance.

Specific heat

Is defined as the amount of heat required to raise a unit mass of the substance through a unit rise in temperature. $C = \frac{Q}{m.\Delta T}$ J/kg.K

Latent heat

The amount of heat transfer required to cause a phase change in unit mass of a substance at a constant pressure & temperature

Points to remember.

1) Heat transfer and work transfer are the energy interactions. A closed system can interact in two ways, by heat transfer and by work transfer.

2) Both heat transfer and work transfer are boundary phenomena.

3) Heat or work is not a property of the system.

4) Heat transfer is the energy interaction due to temperature difference.

5) Both heat & work are path functions and inexact differentials.

1) A single cylinder, double acting, reciprocating water pump has an indicator diagram which is a rectangle 0.075m long & 0.05m high, the indicator spring constant is 147 MPa/m. The pump runs at 500 rpm. The pump cylinder diameter is 0.15m & the piston stroke is 0.2m. Find the rate in kW at which the piston does work on the water.

Soln: $P_m = \frac{a_d}{l_d} \times \text{Spring constant}$

$P_m = \frac{0.075 \times 0.05}{0.075} \times 147 = 7.35 \text{ MPa}$

$W.D = P_m LAN$

$= 7.35 \times 0.2 \times \frac{\pi}{4}(0.15)^2 \times 500$

$\boxed{WD = 12.99 \text{ MJ}}$

2) A turbine is supplied with steam at a gauge pressure of 1.4 MPa. after expansion in the turbine the steam flows into a condenser maintained at a vacuum of 710mm of mercury. The barometric pressure is 772 mm Hg. Express the inlet & exhaust steam pressures in pascals (absolute). Take the density of mercury as 13600 kg/m3.

(i) Absolute Pressure = Gauge Pressure + Atm. pressure

$$= 1.4 + \frac{772 \times 13600 \times 9.81}{100 \times 10^2}$$

$$= 1.4\,MPa + 0.1029\,MPa$$

∴ Ab. pre = 1.503 MPa

(ii) Abs pr = [772 − 710] [B.P − V.P]

$$= \frac{62 \times 13600 \times 9.81}{10^8} = \underline{8.27\,KPa}$$

3) ~~Calculate~~ A milk chilling unit can remove heat from the milk at the rate of 41.87 MJ/h. Heat leaks into the milk from the surroundings at an average rate of 4.187 MJ/h. Find the time required for cooling a batch of 500 kg of milk from 45°C to 5°C. Take cp of milk to be 4.187 kJ/kg.K

Total heat required to be removed

$$= 41.87 - 4.187 \rightarrow 37.683\ MJ/hr$$

Heat contained in milk = 500 × (45−5) × 4.187 kJ.

$$= 83.74\ MJ$$

$$\text{Time required} = \frac{83.74}{37.683} \times 60\,min = 133\ mins.$$

DATE: PAGE:

4) June 2011 06 A system containing 5kg of a substance is stirred with a torque of 1Nm at a speed of 500rpm for 24 hours. The system mean while expands from 1.5m³ to 2m³ against a constant pressure of 5bar. Determine the magnitude and direction of net transfer.

Given : $m = 5kg$, $N = 500rpm$, $V_1 = 1.5m^3$

$T = 1Nm$, $t = 24hrs$, $V_2 = 2m^3$

Heat transferred = $\frac{2\pi NT}{60,000} = \frac{2\pi \times 500 \times 1 \times 24 \times 3600}{60000} = 4,500kJ$

Work done in expansion = $P(V_2 - V_1)$

$= 5\times10^5 (2 - 1.5) = 250kJ$

Net transfer of work = $4500 - 250$

$= 4250kJ$ to the system

5) Dec 2011 06 A cylinder fitted with a piston on which a number of weights are placed. The initial pressure is 200kPa and the initial volume is 0.04m³. Heat is transferred to the system and weights are removed from the piston in such a way that $PV^{1.3}$ = constant describes the relation b/w the pressure & volume during the process. Final volume is 0.1m³. Calculate the work done during the process.

Given : $P_1 = 200\times10^3 N/m^2$, $V_2 = 0.1m^3$

$V_1 = 0.04m^3$, $PV^{1.3} = C$

$$P_1V_1^{1.3} = P_2V_2^{1.3}$$

$$2\times10^5 \times (0.04)^{1.3} = P_2 \times (0.1)^{1.3}$$

$$P_2 = 0.3279 \times 10^5 N/m^2$$

$$W.D = \frac{P_1V_1 - P_2V_2}{n-1} \Rightarrow \frac{P_2V_2 - P_1V_1}{-n+1}$$

$$= \frac{0.3299 \times 10^5 \times 0.3 - 2 \times 10^5 \times 0.04}{1.3 - 1.0} = \underline{\underline{15.69\,kJ}}$$

6) A cylinder contains of 1kg of certain fluid at an initial pressure of 20bar. The fluid is allowed to expand reversible behind a piston according to a law $PV^2=C$ until the volume is doubled. the fluid is then cooled reversibly at constant pressure until the piston regains its original positions, heat is then supply reversibly with the piston firmly locked in position until the pressure rises to the original value calculate the net work done by the fluid for an initial volume of $0.05m^3$.

Dec 2011

10

Given: $P_1 = 20 \times 10^5 N/m^2$, $PV^2 = C$; $V_2 = 2V_1 = 0.1m^3$.

$V_3 = V_1 = 0.05m^3$, $P_2 = P_3 = 5 \times 10^5 N/m^2$

$$P_1V_1^2 = P_2V_2^2$$

$$20 \times 10^5 \times V_1^2 = P_2(2V_1)^2$$

$$P_2 = 5 \times 10^5 N/m^2$$

$$W.D_{1-2} = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{20 \times 10^5 \times 0.05 - 5 \times 10^5 \times 0.1}{2-1}$$

$$\underline{W.D = 50kJ}$$

$$WD_{2-3} = P_2(V_3 - V_2) = 5 \times 10^5 (0.05 - 0.1)$$

$$\underline{WD_{2-3} = -25kJ}$$

$W_{3-1} = 0$ [Isochoric process]

$$\boxed{\therefore W_{net} = 25kJ}$$

7) A certain mass of air is compressed from 1bar, 0.1m³ to 6bar in a piston cylinder device according to $PV^{1.4}$ = const Find the work of compression for air. Had the compression been carried out hyperbolically between the same initial state and the same final pressure as above what would be the work done on air.

gd oy 10//

Given: $P_1 = 1\text{ bar} = 1\times10^5\ N/m^2$, $P_2 = 6\text{ bar} = 6\times10^5\ N/m^2$

$V_1 = 0.1m^3$, $PV^{1.4} = \text{const}$.

$$W.D = \frac{P_1V_1 - P_2V_2}{n-1}$$

$$P_1V_1^n = P_2V_2^n$$

$$1\times\cancel{10^5}\times(0.1)^{1.4} = 6\times\cancel{10^5}\times(V_2)^{1.4}$$

$$\therefore V_2 = 0.0278m^3$$

$$W.D = \frac{1\times0.1\times10^5 - 6\times10^5\times(0.0278)^{1.4}}{1.4-1}$$

$$W.D = \underline{-16.712\ kJ}$$

Hyperbolic (Isothermal).

$$PV = C$$

$$P_1V_1 = P_2V_2$$

$$W.D = P_1V_1 \ln\left(\frac{P_2}{P_1}\right)$$

$$= 1\times10^5\times0.1 \ln\left(\frac{6}{1}\right)$$

$$\boxed{W.D = 17.92kJ}$$

8/ June 2010 A mass of gas is compressed in a quasi static process from 80kPa, 0.1m3 to 0.4MPa, 0.03m3. Assuming that the pressure and volume are related by $PV^n =$ const, find the work interaction during the process. Is it a work producing system (or) work absorbing system.

Given :- $P_1 = 80 kPa$

$V_1 = 0.1 m^3$, $V_2 = 0.03 m^3$.

$P_2 = 0.4 MPa$.

$$P_1 V_1^n = P_2 V_2^n$$

$$80\times10^3 \times 0.1^n = 4\times10^5 \times (0.03)^n$$

$$\left(\frac{0.1}{0.03}\right)^n = 5$$

$$n(\log 0.1 - \log 0.03) = \log 5$$

$$\boxed{\therefore n = 1.337}$$

$$W.D = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{80\times10^3\times0.1 - 0.03\times4\times10^5}{1.337-1}$$

$$\boxed{W.D = -11.87 kJ}$$

-ve sign indicates work is being absorbed by the system

9/ 06. Dec 2010 A balloon of flexible material is to be filled with air from a storage bottle until it has a volume 0.7m3. The atmosp. pressure is 1.013bar. Determine the work done by the system comprising the air initially in the bottle, given that the baloon is light & requires no stretching.

Given :- $V_1 = 0$

$V_2 = 0.7 m^3$

$P = 1.013 bar$

$W.D = P(V_2 - V_1)$.

$= -1.013(0.7 - 0)$.

$W.D = -70.91 kJ$ (W.D on the system)

A spherical baloon has a ~~dbr~~ initial diameter of 25cm and contains air at 1.2bar. Because of heating, the diameter of the baloon increases to 30cm and during the heating process the pressure is found to be proportional to the diameter, calculate the workdone during the process.

WKT $V = \frac{4}{3}\pi r^3$

Given : $D_1 = 0.25m$

$D_2 = 0.30m$

$P_1 = 1.2 \times 10^5 \ N/m^2$

$P \propto D$

$P = CD$

$V = \frac{\cancel{4}}{3} \frac{\pi D^3}{\cancel{8}}$

$V = \frac{\pi D^3}{6}$

$dV = \frac{\pi}{\cancel{6}_2} \times \cancel{3} D^2$

$\boxed{dV = \pi D^2 dD}$

O.O WKT

$W.D = \int_1^2 P dV$

$W.D = \int CD . \pi D^2 dD$

$C = \frac{P}{D} = \frac{1.2 \times 10^5}{0.25}$

$W.D = C \int_{0.25}^{0.30} \pi D^3 dD$

$W.D = \frac{1.2 \times 10^5}{0.25} \left[\pi \left(\frac{0.3^4}{4} \right) - \pi \left[\frac{0.25^4}{4} \right] \right]$

$W.D = \frac{\pi \, 1.2 \times 10^5}{0.25} \left[2.025 \times 10^{-3} - 0.9766 \times 10^{-3} \right]$

$\boxed{W.D = 15.81 J}$ //

14. A gas contained in a cylinder fitted with a piston loaded with a small number of weights is at 1bar pressure and 0.03m³ of volume. The gas is heated until the volume increases to 0.1m³. Calculate the work done by the gas in the following processes. (i) Pressure remains constant, (ii) temperature remains

constant, (iii) $PV^{1.3}=C$ during the process. Show the processes on P-V diagram.

Given :- $P_1 = 1.3$ bar.

$V_1 = 0.03 m^3$

$V_2 = 0.1 m^3$.

i) Const. Pressure. $W.D = P(V_2 - V_1)$.

$W.D = 1.3(0.1 - 0.03) \times 10^5 = \underline{\underline{9.1 kJ}}$.

ii) Const. Tempr. $W.D = P_1 V_1 \ln V_2/V_1$.

$W.D = 1.3 \times 10^5 \times 0.03 \ln (0.1/0.03)$

$W.D = 4.695 kJ$.

iii) Polytropic. $W.D = P_1 V_1 - P_2 V_2 / n-1$.

$P_1 V_1^n = P_2 V_2^n$ or $P_1 V_1^{1.3} = P_2 V_2^{1.3}$.

$1.3 \times 10^5 \times (0.03)^{1.3} = P_2 (0.1)^{1.3}$.

$P_2 = 0.27176 \times 10^5 N/m^2$.

$$W.D = \frac{1.3 \times 10^5 \times 0.03 - 0.27176 \times 10^5 \times 0.1}{0.3}$$

$\boxed{W.D = 3.941 kJ}$

12) To a closed system 150kJ of work is done on it. If the initial volume is $0.6 m^3$ & pressure of the system varies as follows:

$P = (8 - 4V)$, where P is pressure in bar and V is volume in m^3. Determine the final volume and pressure of the system.

J4 / 0'14

Given : $W.D = -150 kJ$.

$V_1 = 0.6 m^3$.

$P = (8 - 4V)$.

$P = ?$ & $V = ?$

$_{1}W.D_{2} = \int P\,dV$:

$$ {}_{1}W_{2} = \int_{V_1}^{V_2} (8-4V)\,dV $$

$$ -150\times10^3 = 10^5 \int_{0.6}^{V_2} (8-4V)\,dV $$

$$ -150\times10^3 = 10^5 \left[8V - \frac{4V^2}{2} \right]_{0.6}^{V_2} $$

$$ -150\times10^3 = 8(V_2-0.6) - 2(V_2^2-0.36) $$

$$ -150\times10^3 = 8V_2 - 4.8 - 2V_2^2 - .72 \times10^5 $$

$$ -1.5 = 8V_2 + 4.8 + 2V_2^2 + 0.72 $$

$$ V_2^2 - 4V_2 + 1.29 = 0 $$

$$ \therefore V_2 = \frac{4 + \sqrt{16 - 4\times1.29}}{2} $$

$$ \boxed{V_2 = 0.353\,m^3} $$

Final pressure $= 8 - 4\times0.353$

$$ \boxed{P = 6.584\ bar} $$

18. A spherical baloon of 1m dia contains a gas at 200kPa. The gas inside the baloon is heated until the pressure reaches 500 kPa. During the process of heating, the pressure of the gas inside the baloon is proportional to the diameter of the baloon. Calculate the work done by the gas inside the baloon. 2016

Given: $D = 1m$ $\quad$ $P \propto D \Rightarrow P = CD$

$P_1 = 200\,kPa$ $\quad$ $200 = C\times1$

$P_2 = 500\,kPa$ $\quad$ $\therefore C = 200\ kPa/m$

$P \propto D$

$W.D = \int$

$P_2 = CD_2$.

$500 = 200\, D_2 \quad ∴ \boxed{D_2 = 2.5\,m}$

Volume of sphere : $V = \frac{\pi D^3}{6}$.

$dV = \frac{\pi}{\cancel{6}\,2} \times \cancel{3} D^2 dD$.

∴ $W.D = \int P\,dV$.

$W.D = \int CD \times \frac{\pi}{2} D^2 dD$.

$W.D = \frac{C\pi}{2} \int_{D_1}^{D_2} D^3\, dD$.

$W.D = \frac{200 \times \pi}{2} \left[\frac{D_2^4}{4} - \frac{D_1^4}{4}\right]$

$W.D = \frac{200 \times \pi}{8} \left[D_2^4 - D_1^4\right]$

$W.D = \frac{200 \times \pi}{8} \left[2.5^4 - 1.^4\right]$

$\boxed{W.D = 2989\, kJ}$

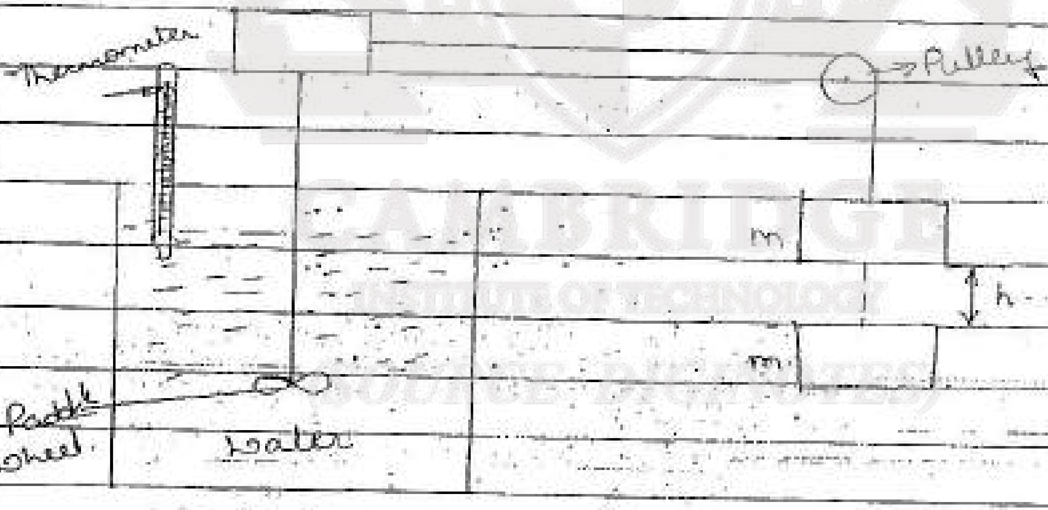
MODULE -2

FIRST LAW OF THERMODYNAMICS

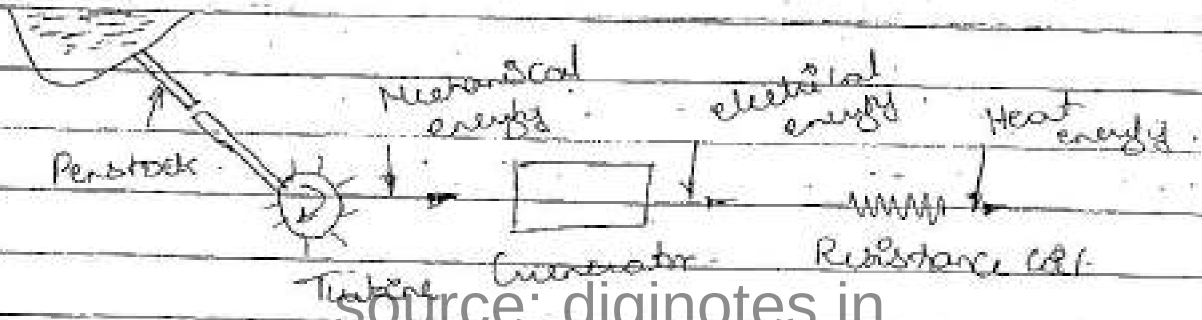
Introduction:

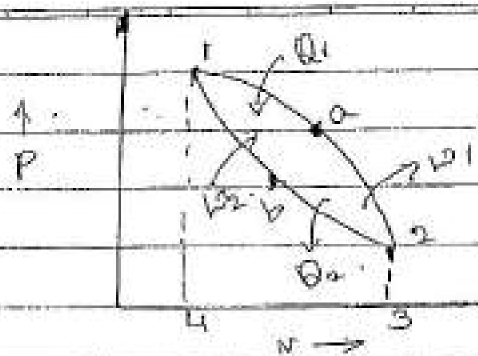
The first law of thermodynamics gives the law of conservation of energy principle. Based on the energy transfer in the form of work and heat, the first law relation for closed system is introduced. The concepts of internal energy and enthalpy are defined.

First law of thermodynamics for a closed system undergoing a cycle. or Joule's Paddle-wheel experiment



$Q \propto W$ or $Q = W/J$





E.g.: The heat of fuel input to an internal combustion engine can be accounted for output in the form of mechanical energy at the shaft and heat loss to cooling medium and surroundings etc.

* There occurs conversion of potential energy into kinetic energy when a fluid flows through a pipeline.

First law of Thermodynamics

A process is cyclic if the initial and final states of the system executing the process are identical.

A system represented by state point 1 undergoes a process 1-a-2 and comes back to initial state following the path 2-b-1. All properties of the system are restored, when the initial state is reached.

(i) area 1-a-2-3-4-1 represents the work w_1 done by the system during expansion process 1-a-2.

(ii) area 2-3-4-1-b-2 represents the work w_2 supplied to the system during compression process 2-b-1.

(iii) area 1-a-2-b-1 represents the net work $(w_1 - w_2)$ delivered by the system.

∴ The first law of thermodynamics for a cyclic process can be expressed as

$$w_1 - w_2 = Q_1 - Q_2$$

$$\oint \delta Q = \oint \delta W$$

cyclic integral of heat $\oint \delta Q$ is equal to cyclic integral of work $\oint \delta W$, both forms of energy expressed in same units

$$\oint (\delta Q - \delta W) = 0$$

It is to be noted that though both δQ and δW are path functions, their difference $(\delta Q - \delta W)$ is a point function as the integral $\oint (\delta Q - \delta W)$ is zero.

Corollaries of first law

i. when a system executes a process, the net heat interaction equals the net work interaction plus change in stored energy

$$\delta Q = \delta W + dE$$

It is stated as " There exists a property of a closed system such that a change in its value during any change of state is given by the difference b/w the heat supplied and work done"

$$dE = \delta Q - \delta W$$

The property E is called energy of the system. This energy is within the system, it increases or decreases with change of state

E is a property, it can be internal energy, kinetic energy, potential energy, electrical energy, chemical energy & magnetic energy are some of the important modes of energy

ii) For an isolated system, both heat and work interactions are absent $(\delta Q = 0, \delta W = 0)$

$$dE = 0 \text{ (or) } E = \text{constant}$$

(un-interrupted)

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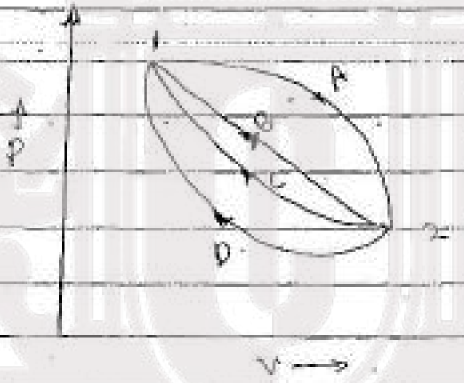
(iii) A perpetual motion machine of the first kind is impossible.

It is an imaginary device which produces a continuous supply of work without absorbing any energy from the surroundings (or from other system. It violates the first law of thermodynamics.

Stored energy - A property of the system.



Consider a closed system which changes from state 1 to state 2 by path A and returns back to original state 1 by any one of the following paths:

(i) along path 2B1

(ii) along path 2C1

(iii) along path 2D1

Applying first law for the cyclic process

$$1-A-2-B-1$$

$$\oint (\delta Q - \delta W) = 0$$

$$\text{(or)} \int_{1(A)}^{2} (\delta Q - \delta W) + \int_{2(B)}^{1} (\delta Q - \delta W) = 0 \quad \text{----} (i)$$

For the cycle 1-A-2-C-1

$$\int_{1(A)}^{2}(\delta Q-\delta W)+\int_{2(C)}^{1}(\delta Q-\delta W)=0 \quad \ldots\ldots(ii)$$

and for the cycle 1-A-2-D-1

$$\int_{1(A)}^{2}(\delta Q-\delta W)+\int_{2(D)}^{1}(\delta Q-\delta W)=0 \quad \ldots\ldots(iii)$$

Comparing eqns (i), (ii) & (iii),

$$\int_{2(B)}^{1}(\delta Q-\delta W)=\int_{2(C)}^{1}(\delta Q-\delta W)=\int_{2(D)}^{1}(\delta Q-\delta W).$$

Since B, C & D represent arbitrary paths between the state point 2 and state point 1. it can be concluded that the integral

$$\int_{2}^{1}(\delta Q-\delta W)$$

(i) remains the same irrespective of the path along which the system proceeds.
(ii) is solely dependent on the initial and final states of the system, is a point function and hence a property.

$\int_{2}^{1}(\delta Q-\delta W)$ is called energy of the system & is given the symbol E.

Energy is a property of the system, its differential is exact and is denoted by dE

$\therefore \delta Q-\delta W=dE$; $\delta Q=\delta W+dE$

The energy E is an extensive property.

The specific energy, $e=\frac{E}{m}$, is an intensive property.

In a reversible non-flow process.

$$\delta W = p\,dV$$

$$\therefore \delta Q = p\,dV + dE$$

The energy E includes all forms of energy in the system.

$$E = U + K.E + P.E + C.E + E.E + M.E + \ldots\ldots$$

where U is associated with molecular motion and is stored in the molecules and atomic structure of the substance.

Potential Energy.

The energy which a system possesses by virtue of its position in a force field is called potential Energy.

$$dPE = mg\,dZ$$

Kinetic Energy.

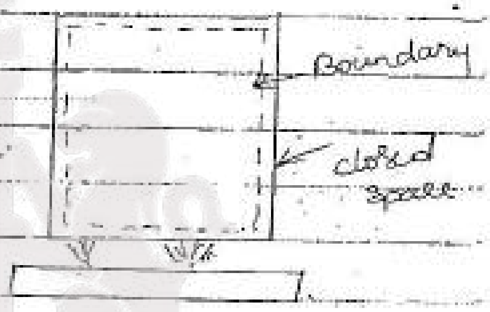
The energy which a system possesses owing to its motion is called kinetic Energy.

$$KE = \frac{m\bar{V}^2}{2} \quad \text{(or)} \quad dKE = m\bar{V}\,d\bar{V}$$

Constant volume (or isochoric process.

A change in the state of a system at constant volume is called isochoric process. An isochoric process results when the gas system is heated (or cooled in an enclosed space.

During the process, both the pressure & temperature increase when heat is supplied to the system, & decrease when heat is rejected by the system.



For the initial state 1:

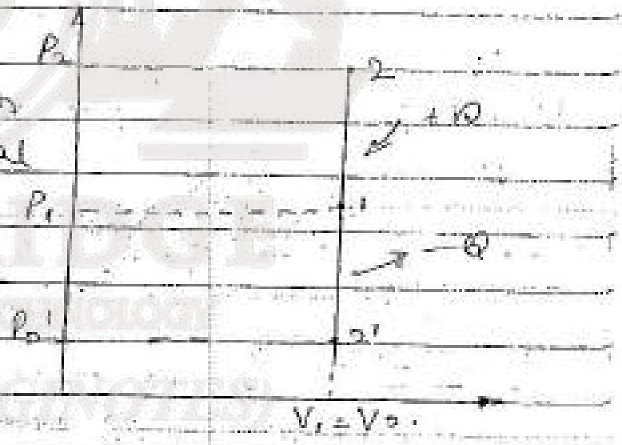
$P_1V_1 = mRT_1 \rightarrow (i)$

$P_2V_2 = mRT_2$ (State 2) $\rightarrow (ii)$

As $V_1 = V_2$, dividing (i) by (ii)

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \text{ or } \frac{P}{T} = \text{Const}$$

"Charle's law"



Since there is no expansion of gas ($dV=0$), no mechanical work is done on (or by the system.

$\therefore \delta Q = \delta W + dU$

$\delta W = 0$.

all the heat is utilized to change the internal energy of the system.

Heat added during a constant volume process is given by

$\delta Q = mC_v(T_2 - T_1)$

$dU = \delta Q = mC_v(T_2 - T_1)$

for unit mass of gas, $dU = \delta Q = C_v dT$ or $C_v = \frac{du}{dT}$

$\therefore$ Specific heat at constant volume C_v is defined as the rate of change of specific internal energy w.r.to change of temp^r when it is a constant volume process.

constant pressure or Isobaric process.

During an isobaric process, both the volume & temperature change. The addition of heat causes the temperature of the gas and its volume to increase, the rejection of heat causes it to reduce.

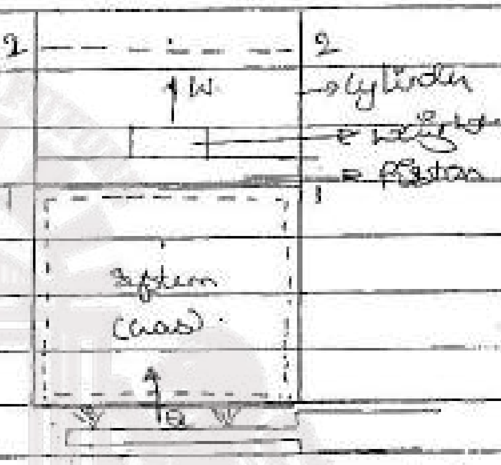
For initial state,

$P_1V_1 = mRT_1 \rightarrow (i)$

$P_2V_2 = mRT_2 \rightarrow (ii)$

$P_1 = P_2$, dividing (i) & (ii)

$\frac{V_1}{V_2} = \frac{T_1}{T_2}$ or $\frac{V}{T} = \text{const}$



Mechanical work

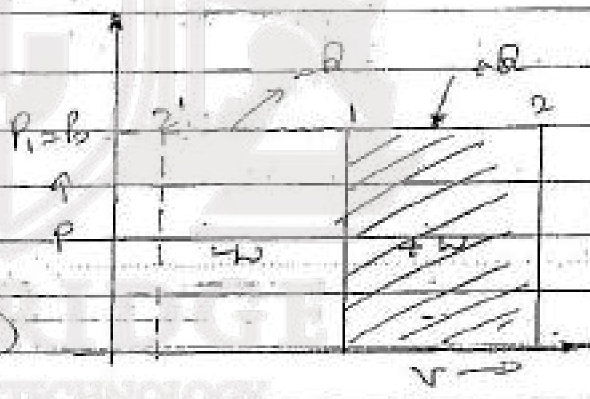
$W_{1-2} = \int_1^2 PdV = P(V_2 - V_1)$.

For a non-flow process

$\delta Q = \delta W + dU$

$Q_{1-2} = P(V_2 - V_1) + (U_2 - U_1)$

$= (U_2 + P_2V_2) + (U_1 + P_1V_1)$



where U+PV is a property i.e enthalpy.

$\therefore Q_{1-2} = H_2 - H_1$

$\therefore$ The amount of heat supplied to a system during an isobaric process equals the difference in enthalpies at the end and at the beginning of the process.

$H_2 - H_1 = Q_{1-2} = mC_p(T_2 - T_1)$.

for a unit mass of the gas,

$dh = \delta q = C_p dT$ & $C_p = \frac{dh}{dT}$

Specific heat at constant pressure Cp may be defined as the rate of change of specific enthalpy with respect to change of temperature when the process is carried out at constant pressure.

Specific workdone,

$$w_{1-2} = \int P dv = P(V_2 - V_1).$$

Since $P_1 = P_2 = P$

$P_1V_1 = RT_1$ and $P_2V_2 = RT_2$

$w_{1-2} = R(T_2 - T_1)$ ∴ $R = \frac{w_{1-2}}{(T_2 - T_1)}$.

Relation b/w specific heats & gas constant.

Consider the system of an ideal gas which satisfies the relations

$PV = RT$; $C_v = du/dT$ & $C_p = dh/dT$

Enthalpy is the sum of internal energy & pressure volume.

$h = u + PV \Rightarrow u + RT$

differentiating, w.r.t. T.

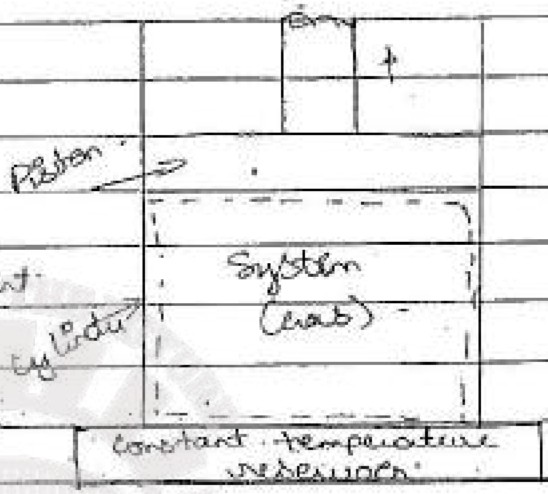
$dh = du + RdT$

$\frac{dh}{dT} = \frac{du}{dT} + R.$

$C_p = C_v - R$ ∴ $C_p - C_v = R$.

The value of Cp is greater than Cv, because the heat supplied to a gas at constant pressure is utilized for two purposes, for doing the external work and for increasing the internal energy. The ratio of C_p/C_v is ~~dety~~ γ and is called adiabatic index or adiabatic constant.

Isothermal process (Constant temperature)



for initial state:

$P_1 V_1 = mRT_1 \rightarrow (i)$

$P_2 V_2 = mRT_2 \rightarrow (ii)$

$T_1 = T_2$, ∴ (i) by (ii)

$P_1 V_1 = P_2 V_2$ (or $PV = \text{constant}$

(Boyle's Law)

Work done

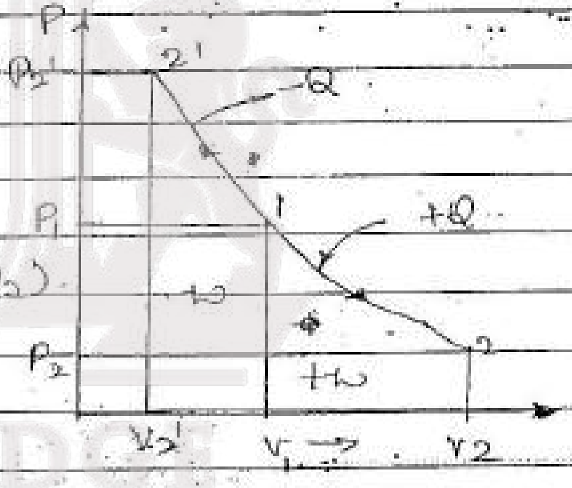
$W_{1-2} = \int_1^2 P\,dv$

$PV = \text{const } C, \quad P = \frac{C}{V}$

$\therefore W_{1-2} = C \int_1^2 \frac{dv}{V}$

$= C \log_e \frac{V_2}{V_1} = P_1 V_1 \log_e \frac{V_2}{V_1}$

$= P_1 V_1 \log_e \frac{P_1}{P_2} \quad (\because P_1 V_1 = P_2 V_2)$



change in internal Energy

$\Delta U = mC_v (T_2 - T_1) = 0$

$Q_{1-2} = W_{1-2}$

In an isothermal process all the heat received by the gas is utilized for doing mechanical work.

Polytropic process

work is done by the working fluid as well as heat is absorbed

$Q_{1-2} = W_{1-2} + dU$

$= \frac{P_1 V_1 - P_2 V_2}{n-1} + C_v (T_2 - T_1)$

$= \frac{R(T_1 - T_2)}{n-1} + C_v (T_2 - T_1)$

$= \frac{(C_p-C_v)(T_1-T_2)}{n-1} + C_v(T_2-T_1)$ (∵ $R = C_p - C_v$).

$= (T_1-T_2)\left[C_v - \frac{C_p-C_v}{n-1}\right]$ $(T_1-T_2)\left[\frac{C_p-C_v}{n-1} + C_v\right]$.

$= \frac{C_p-nC_v}{1-n}(T_2-T_1)$. $= \frac{C_p-nC_v}{1-n}(T_2-T_1)$.

$= C_n(T_2-T_1)$.

where $C_n = \frac{C_p-nC_v}{1-n}$ is called the polytropic specific heat.

Substituting $\frac{C_p}{C_v} = \gamma$, we get

∴ $C_n = \frac{\gamma-n}{1-n}C_v = -\left[\frac{\gamma-n}{n-1}C_v\right]$

C_n is negative when $\frac{\gamma-n}{1-n}$ is positive, i.e when $\gamma > n > 1$.

Process	work o/p	Heat received
Isochoric	$W_{1-2} = 0$	$Q_{1-2} = U_2 - U_1$ $= mC_v(T_2-T_1)$
Isobaric	$W_{1-2} = p\int dV$	$Q_{1-2} = H_2 - H_1$ $= mC_p(T_2-T_1)$.
Isothermal	$W_{1-2} = P_1V_1\log_e\frac{V_2}{V_1}$	$Q_{1-2} = W_{1-2}$
Adiabatic	$W_{1-2} = \frac{P_1V_1-P_2V_2}{\gamma-1}$ $= \frac{mR(T_1-T_2)}{\gamma-1}$	$Q_{1-2} = 0$
Polytropic ($PV^n = \text{const}$)	$W_{1-2} = \frac{P_1V_1-P_2V_2}{n-1}$	$Q_{1-2} = W_{1-2} + (U_2-U_1)$. $= \gamma - n \times W_{1-2}$

Relation b/w P, V, T

WKT $PV^\gamma = C$

i.e $P_1V_1^\gamma = P_2V_2^\gamma = C$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma \quad \ldots\ldots (1)$$

from gas equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

(or) $\frac{P_2}{P_1} = \frac{V_1}{V_2} \times \frac{T_2}{T_1}$

Sub this in eqn (1), we get

$$\frac{V_1}{V_2} \times \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^\gamma$$

$$\frac{T_2}{T_1} = \frac{\left(\frac{V_1}{V_2}\right)^\gamma}{\left(\frac{V_1}{V_2}\right)} = \left(\frac{V_1}{V_2}\right)^\gamma \cdot \left(\frac{V_2}{V_1}\right)$$

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \quad \ldots\ldots (2)$$

Again from gas relation

$$\frac{V_1}{V_2} = \frac{P_2}{P_1} \times \frac{T_1}{T_2}$$

Sub this in eqn (1), we get

$$\frac{P_2}{P_1} = \left(\frac{P_2}{P_1} \times \frac{T_1}{T_2}\right)^\gamma$$

$$\left(\frac{T_1}{T_2}\right)^\gamma = \frac{\left(\frac{P_2}{P_1}\right)}{\left(\frac{P_2}{P_1}\right)^\gamma} = \left(\frac{P_2}{P_1}\right)\left(\frac{P_2}{P_1}\right)^{-\gamma} = \left(\frac{P_2}{P_1}\right)^{1-\gamma}$$

$$\frac{T_1}{T_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1-\gamma}{\gamma}}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{-\left(\frac{1-\gamma}{\gamma}\right)} \quad \text{(or)} \quad \boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}$$

Flow process and control volume

A flow process constitutes an open system through which the working fluid enters and leaves from the surface of a system. There also exists energy interactions in the form of heat and work, internal energy, gravitational potential energy & kinetic energy. ex: engines, turbines pumps and compressors, boilers and heat exchangers etc.

An imaginary envelope that exists around an equipment, this envelope is called control surface. and the space bounded is called control volume. The control surface is fixed in position and does not change its shape or orientation during the flow process. fig (a)

Flow work or flow energy.

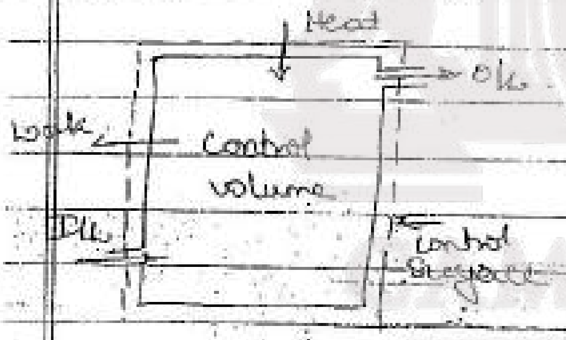


fig (a)

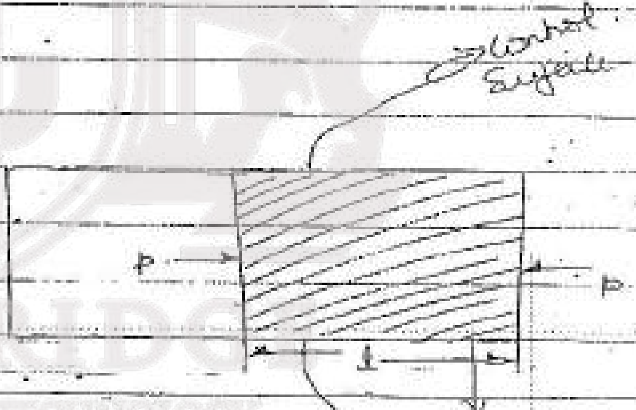


fig (b)

From fig (b) flow energy or flow work refers to the work which a fluid does when it moves in or out of any passage.

Consider a unit mass of fluid, which is entering the control volume through a channel of c/s A. The force acting on the element is pA. 'l' is the distance through which the fluid mass is pushed in,

∴ workdone = force × distance moved

pAl

∴ Flow work per unit mass = pv.
where P ⇒ Pressure & v ⇒ Specific volume of the flowing fluid.

Steady & unsteady flow process.

A flow process is said to be steady when the fluid parameters at any point of the control volume remain constant with respect to time. ∴ The quantities like velocity, pressure, temperature etc are functions of location & do not vary with time.

A steady flow process follows these conditions in control volume:

- no change in the mass of ~~dry~~ fluid in the control volume. mass inflow equals the mass outflow.
- the state of fluid at any point is same at all times
- the heat & work interactions at the control surface are at a constant rate

Flow process is unsteady when the conditions vary with respect to time.

$\frac{dP}{dt} = 0$ (for steady process) $\frac{dP}{dt} \neq 0$ (for unsteady process)

Steady flow energy equation.

Consider of flow of fluid through a system, in which one stream of fluid enters and another stream leaves the control volume. There is no accumulation of mass & energy within the control volum and the properties at any location within the control volume are steady with time. Sections 1-1 & 2-2 are the entrance and exit of the fluid across the control surface.

$A_1, A_2 \rightarrow$ c/s of stream, m^2

$w_1, w_2 \rightarrow$ mass flow rate, kg/s.

$p_1, p_2 \rightarrow$ pressure, absolute, N/m^2

$v_1, v_2 \rightarrow$ specific volume, m^3/kg

$u_1, u_2 \rightarrow$ specific internal energy, J/kg

$V_1, V_2 \rightarrow$ velocity, m/s

$Z_1, Z_2 \rightarrow$ elevation above datum, m

$\frac{\delta Q}{\delta T} \rightarrow$ net rate of heat transfer through the control surface, J/s

$\frac{\delta W}{\delta t} \rightarrow$ net rate of work transfer through the control surface, J/s.

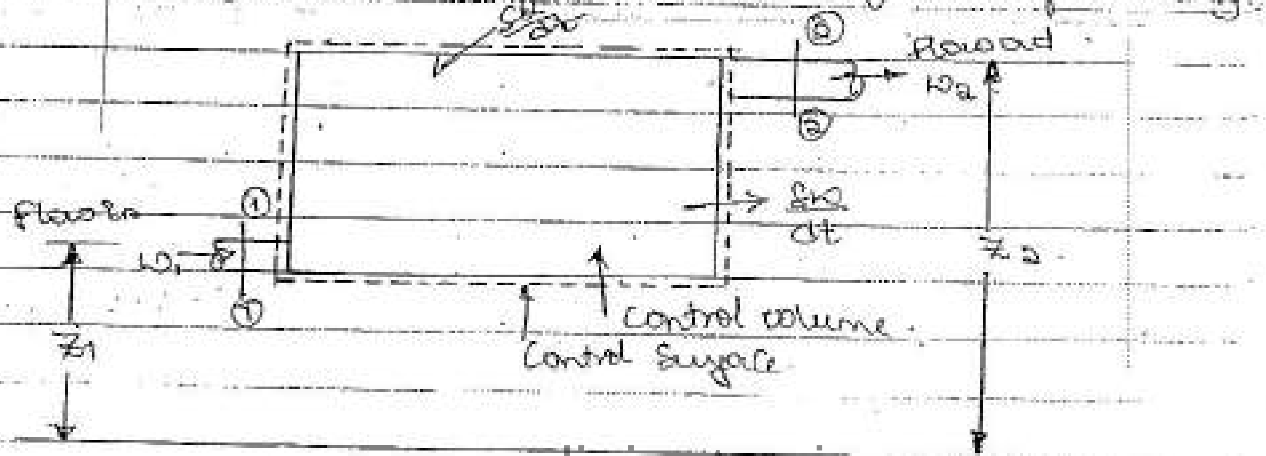
1 & 2 are the inlet and exit for the system

By conservation of mass, if there is no accumulation of mass within the control volume, the mass flow rate entering must equal the mass flow rate leaving,

$$w_1 = w_2$$

$$\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

This equation is known as the equation of continuity.



The work transfer is of two types, the external work and the flow work. The external work are such as shear, shaft work, stirring and electrical work. Flow work is the displacement work done by the fluid of mass dm_1 at the inlet and that of mass dm_2 at the exit section. i.e $(-P_1 v_1 dm_1)$ & $(+P_2 v_2 dm_2)$.

∴ Total work transfer is given by,

$$W = W_x - P_1 v_1 dm_1 + P_2 v_2 dm_2$$

$$\frac{\delta W}{\delta t} = \frac{\delta W_x}{\delta t} - P_1 v_1 \frac{dm_1}{dt} + P_2 v_2 \frac{dm_2}{dt}$$

or $$\frac{\delta W}{\delta t} = \frac{\delta W_x}{dt} - w_1 P_1 v_1 + w_2 P_2 v_2$$

the total rate of flow of all energy streams entering the control volume must equal the total rate of flow of all energy streams leaving the control volume.

$$w_1 e_1 + \frac{\delta Q}{dt} = w_2 e_2 + \frac{\delta W}{\delta t}$$

Substituting for $\frac{\delta W}{dt}$,

$$w_1 e_1 + \frac{\delta Q}{dt} = w_2 e_2 + \frac{\delta W}{dt} - w_1 P_1 v_1 + w_2 P_2 v_2$$

$$w_1 e_1 + w_1 P_1 v_1 + \frac{\delta Q}{dt} = w_2 e_2 + w_2 P_2 v_2 + \frac{\delta W}{dt}$$

where e_1 and e_2 refer to the energy carried ~~out~~ into or out of the control volume with unit mass of fluid.

The specific energy e is given by

$$e = e_k + e_p + u = \frac{V^2}{2} + Zg + u$$

Substituting the expression for e.

$$w_1\left[\frac{V_1^2}{2} + Z_1 g + u_1\right] + w_1 P_1 v_1 + \frac{\delta Q}{dt} =$$

$$+ \frac{u_2}{...}$$

$$m\left[P_1 v_1 + \frac{V_1^2}{2} + g Z_1\right) \ldots m \left(P_2 v_2 + \frac{V_2^2}{2} + g Z_2\right) + W$$

$$w_2\left[\frac{V_2^2}{2}+Z_2g+u_2\right]+w_2p_2v_2+\frac{\delta W_x}{dt}$$

(or) $$w_1\left[h_1+\frac{V_1^2}{2}+Z_1g\right]+\frac{\delta Q}{dt}=w_2\left[h_2+\frac{V_2^2}{2}+Z_2g\right]+\frac{\delta W_x}{dt}$$

where $h = u+pv$.

Since, $w_1 = w_2$, let $w = w_1 = w_2 = \frac{dm}{dt}$.

diving by dm/dt.

$$h_1+\frac{V_1^2}{2}+Z_1g+\frac{\delta Q}{dm}=h_2+\frac{V_2^2}{2}+Z_2g+\frac{\delta W_x}{dm}$$

These are known as steady flow energy equations (SFEE) for a single stream of fluid entering and a single stream of fluid leaving the contol volume.

Property Relations for Energy Equations.

WKT $h = u+pv$ (dif^n.)

$dh = du + vdp + pdv$

But $dQ = du + pdv$ (for closed system)

$du = dQ - pdv$

Substituting

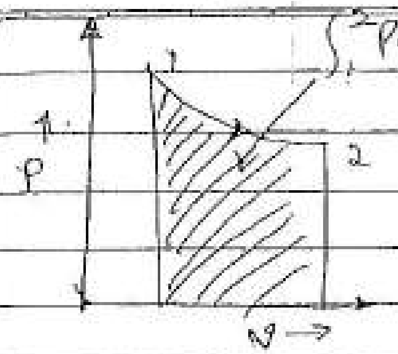
$dh = dQ - \cancel{pdv} + vdp + \cancel{pdv}$

$dh = dQ + vdp$

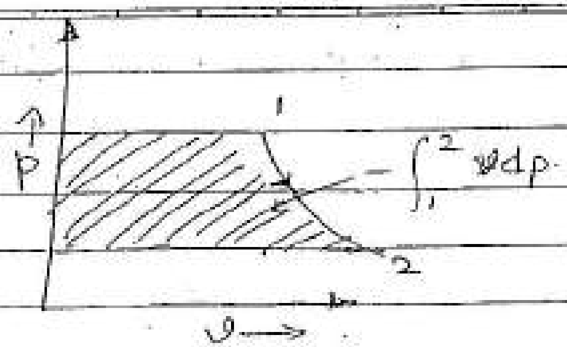
$vdp = dh - dQ$.

$-\int_1^2 vdp =$ ~~dh - dQ~~ $Q - \Delta h$.

where $-\int_1^2 vdp$ represented on P-v diagram,



for non-flow process



for flow process.

(i) For Steady flow constant pressure process.

$$W = -\int_1^2 v.dp = 0 \quad [\because dp = 0]$$

(ii) For Steady flow constant volume process

$$W = -\int_1^2 Vdp = -V(P_2 - P_1) = V(P_1 - P_2)$$

$$W = V(P_1 - P_2)$$

(iii) Steady flow constant temperature process

$$PV = P_1V_1 = P_2V_2 = C$$

$$W = -\int_1^2 Vdp$$

$$= -\int_1^2 \frac{C}{P} dp$$

$$= -C\int_1^2 \frac{dP}{P} = -C\log_e P\Big|_1^2$$

$$= -C\log_e \frac{P_2}{P_1} = C\log_e \frac{P_1}{P_2}$$

$$W = P_1V_1 \log_e\left(\frac{P_1}{P_2}\right)$$

(iv) $W = RT_1 \log_e \frac{P_1}{P_2}$ / $W = RT_1 \log_e \frac{v_2}{v_1}$

Polytropic process.

$Pv^n = \text{const}$; $v = \left(\frac{C}{P}\right)^{1/n}$.

$$-\int_1^2 v\,dp = -\int_1^2 \left(\frac{C}{P}\right)^{1/n} dp$$

$$= -C^{1/n}\left[\frac{P^{-\frac{1}{n}+1}}{-\frac{1}{n}+1}\right]_1^2$$

$$= -\frac{n}{n-1}\left[P^{\frac{n-1}{n}} \cdot C^{\frac{1}{n}}\right]_1^2$$

$$= -\frac{n}{n-1}\left[P^{\frac{n-1}{n}}\, P^{\frac{1}{n}}\, v\right]_1^2$$

$$= -\frac{n}{n-1}\left[Pv\right]_1^2$$

$$= \frac{n}{n-1}\left[P_1v_1 - P_2v_2\right]$$

$$= \frac{n}{n-1} R(T_1 - T_2)$$

$$W.D = \frac{n}{n-1} R(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2} + g(Z_1 - Z_2).$$

During polytropic expansion $T_1 > T_2$ and so W.D is +ve which implies that work is done by the system

The work interaction during reversible adiabatic process $pv^\gamma = \text{const}$, by simply replacing polytropic index n by adiabatic index γ.

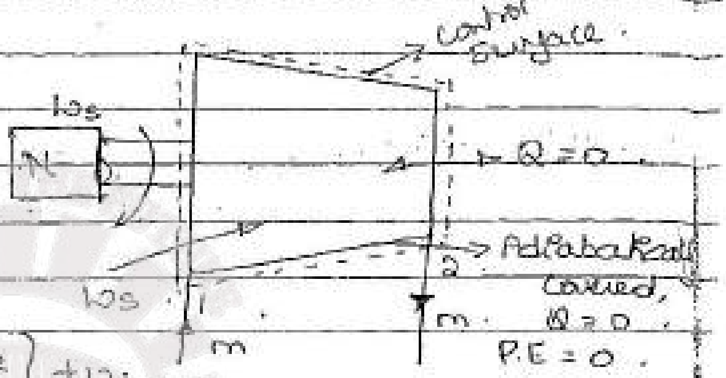
$$W.D = \frac{\gamma}{\gamma-1}(P_1V_1 - P_2V_2) \text{ or } \frac{\gamma}{\gamma-1} R(T_1 - T_2)$$

Applications of Steady flow energy equation

Work absorbing Systems.

Rotary Compressor.

$$Q = \left[h_2 + \frac{V_2^2}{2} + Z_2 g\right] - \left[h_1 + \frac{V_1^2}{2} + Z_1 g\right] + w$$



$$Q = \left[h_2 + \frac{V_2^2}{2}\right] - \left[h_1 + \frac{V_1^2}{2}\right] + w.$$

$$0 = \left[h_2 - h_1 + \frac{V_2^2}{2} - \frac{V_1^2}{2}\right] + w.$$ (work is done on the System).

$w_{1-2} = [h_2 - h_1]$ if K.E is also neglected.

If it is not adiabatically covered, then.

$$-Q_{1-2} = [h_2 - h_1] - w_{1-2}$$

$$w_{1-2} = [h_2 - h_1] + Q_{1-2} \;//.$$

Rotary compressors are the m/cs which develop pressure and have a rotor as primary element. Rotary compressors are employed where high efficiency, medium pressure rise & large flow rate are primary considerations.

The features of rotary compressor are

- Shaft work is done on the System, w is negative.
- Potential energy changes are negligible.
- Heat is lost from the surroundings, Q is negative.

Reciprocating Compressor.

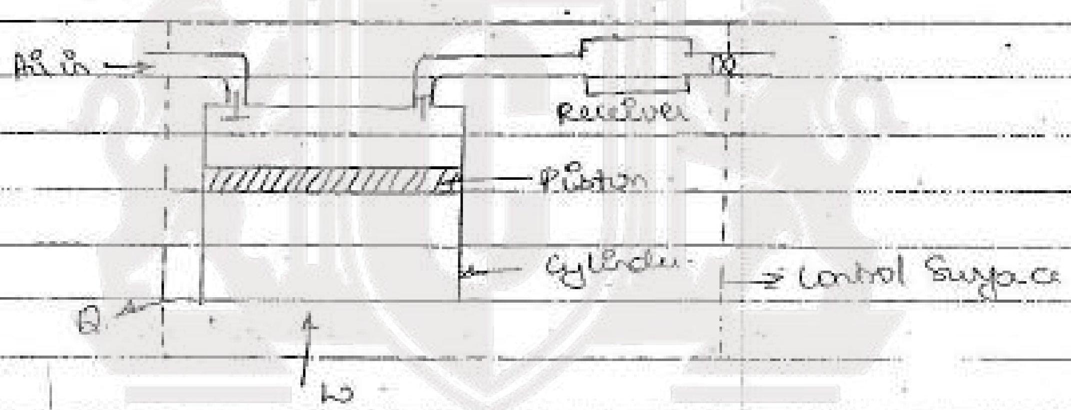
A reciprocating compressor is a device which is used for increasing the pressure of a fluid (gas mainly) and has a piston cylinder mechanism. The unit sucks in definite quantity of fluid, compresses through a required pressure ratio & then delivers.

They are used when small quantity of fluid with high pressure is required

The features of reciprocating compressors are:

- Potential & kinetic energy changes are negligible.
- Shaft work is done on the system, w_s is negative.
- Heat is lost from the system and Q is negative

$$\therefore \; w_s = Q + m(h_2 - h_1).$$



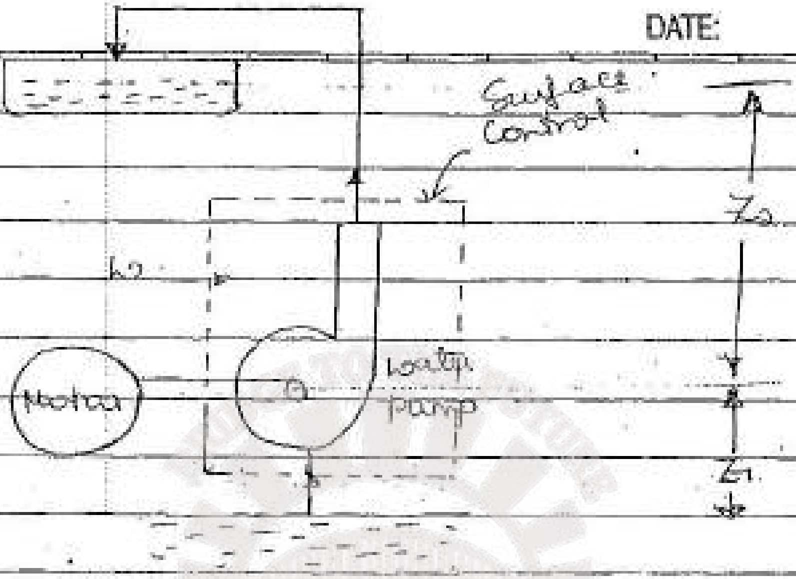
Centrifugal water pump.

~~A~~ pump is a unit that transfers the mechanical energy of a motor or an engine into the potential energy of a ~~motor~~ liquid. ~~to an engine. Into the potential~~ Centrifugal pumps pumps the liquids from lower level to higher level by rotating motion of one or more rotating wheels called impellers.

- $Q = 0$, $du = 0$ (there is no change in temp. of water)

w is negative, $\therefore$

$$m\left(p_1 v_1 + \frac{V_1^2}{2} + g z_1\right) = m\left(p_2 v_2 + \frac{V_2^2}{2} + g z_2\right) - w$$

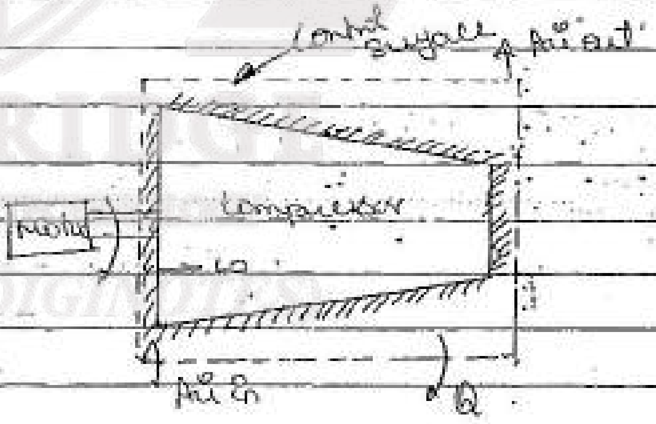


work developing System

Turbine (Steam on gas)

A turbine is a device for obtaining work from a flow of fluid expanding from high pressure to low pressure. The output from the turbine may be used to run a generator & produce electric power.

Features of Turbine



- * negligible flow ; $V=0$
- * negligible P.E ; $Z=0$
- * no heat transfer as walls are insulated ; $Q=0$.

$\therefore m h_1 = m h_2 + W_s$

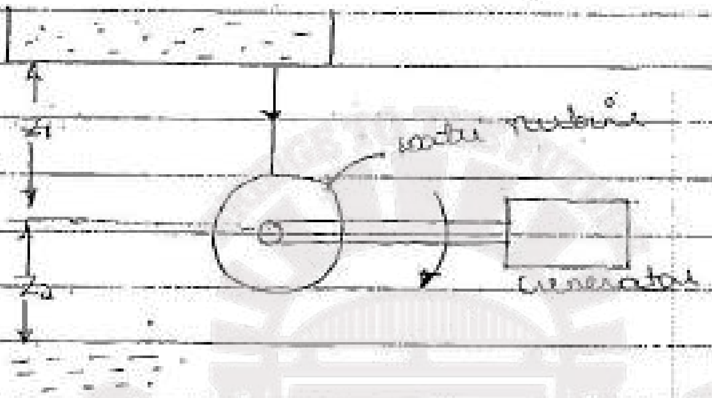
$W_s = m(h_1 - h_2)$.

Hydraulic Turbine

A hydraulic turbine is a prime mover, it uses the potential & kinetic energy of water and converts it into mechanical energy

- * $Q=0$.
- * $du=0$ (no change in temperature)

$$\therefore m\left(P_1v_1 + \frac{V_1^2}{2} + gZ_1\right) = m\left(P_2v_2 + \frac{V_2^2}{2} + gZ_2\right) + W$$

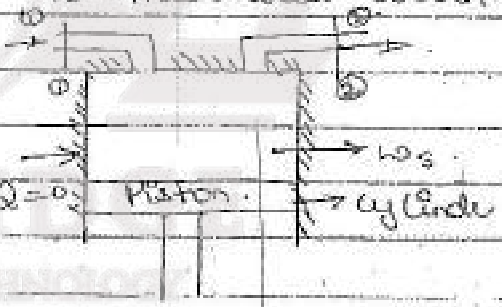


I.C. Engine

Internal combustion engines are the setup where combustion of fuel takes place in the cylinder. Here chemical energy on combustion is converted to mechanical work.

- $W_s = +ve$
- $Q = 0$ (Insulated)
- KE & PE $= 0$

$\therefore W_s = m[h_1 - h_2]$



Non work developing and Non work absorbing system

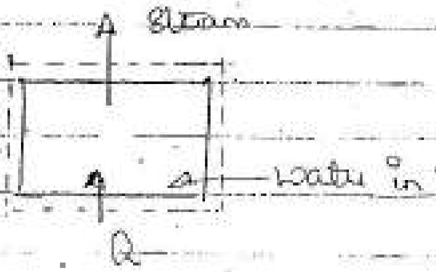
Boiler

It is an equipment used for generation of steam.

- $W_s = 0$
- $KE = 0$
- $PE = 0$

$Q = +ve$ (heat added)

$\therefore Q = m[h_2 - h_1]$

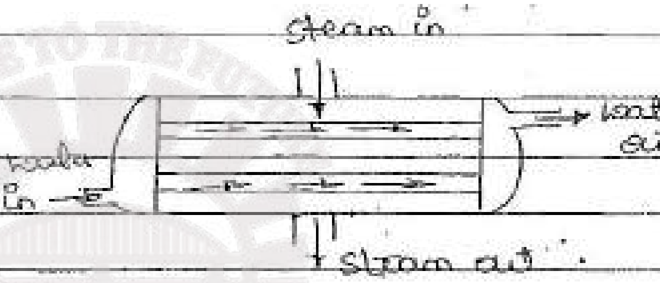


Heat Exchanger (Condenser)

A heat exchanger is a device in which heat is transferred from one fluid to another. A Steam Condenser is also a heat exchanger in which steam loses heat as it passes over the tubes through which water is flowing.

- * $W_s = 0$
- * KE & PE = 0
- * $Q = 0$ (Insulated

From Energy balance



Heat lost by Steam

= heat gained by water

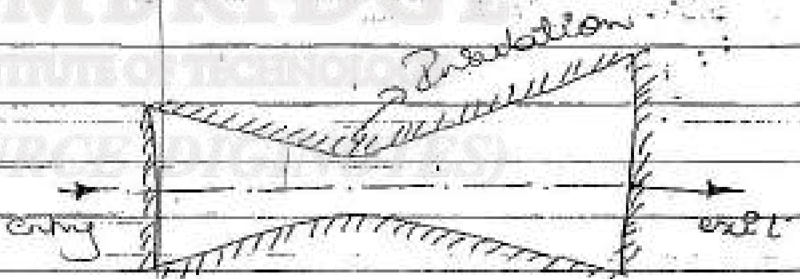
$m_s(h_1 - h_2) = m_w(h_4 - h_3)$.

Some other Systems

Steam nozzle: (Diffuser).

A nozzle is a device used for increasing the velocity of a steadily flowing stream by causing a pressure drop. Enthalpy drops & flow of fluid is increased.

- $W_s = 0$
- * $Q = 0$.
- * $Z_1 - Z_2 = 0$



$$\therefore \frac{V_2^2 - V_1^2}{2} = (h_1 - h_2)$$

from continuity equation;

$$m = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2} \;;\; V_1^2 = V_2^2 \left[\frac{A_2 v_1}{A_1 v_2}\right]^2$$

$$\therefore V_2^2 - V_1^2 \left[\frac{A_2 v_1}{A_1 v_2}\right]^2 = 2(h_1 - h_2).$$

$$V_2 = \left[\frac{1}{1-(A_2 v_1 / A_1 v_2)^2}\right]^{1/2} \sqrt{2(h_1-h_2)}\ .$$

If inlet velocity ~~or~~ is very small compared to exit, we get $V_2 = \sqrt{2(h_1-h_2)}$.

If working fluid is perfect gas, then

$$Pv = RT \ ; \ h = C_pT \ ; \ \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \frac{V_2^2 - V_1^2}{2} = \text{change in K.E}$$

$$= (h_1 - h_2)$$

$$= C_p(T_1 - T_2)$$

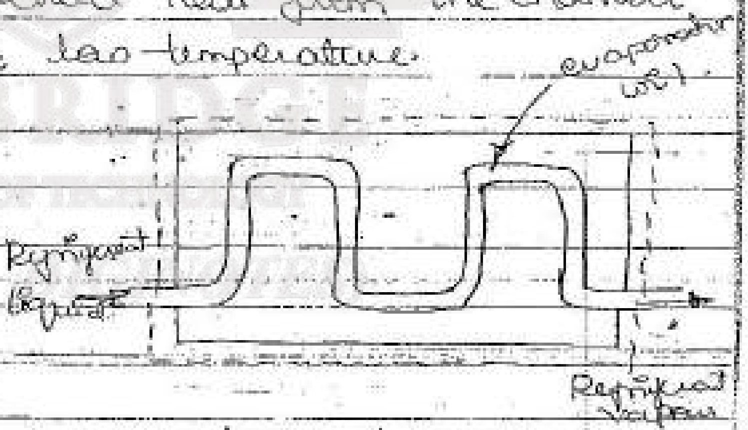
$$= C_pT_1\left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

Evaporator

An evaporator is the component of a refrigeration system, and is used to extract heat from the chamber which is to be kept at low temperature.

* $W_s = 0$

* $KE \Rightarrow PE = 0$



$$\therefore m(h_1) + Q = m(h_2)$$

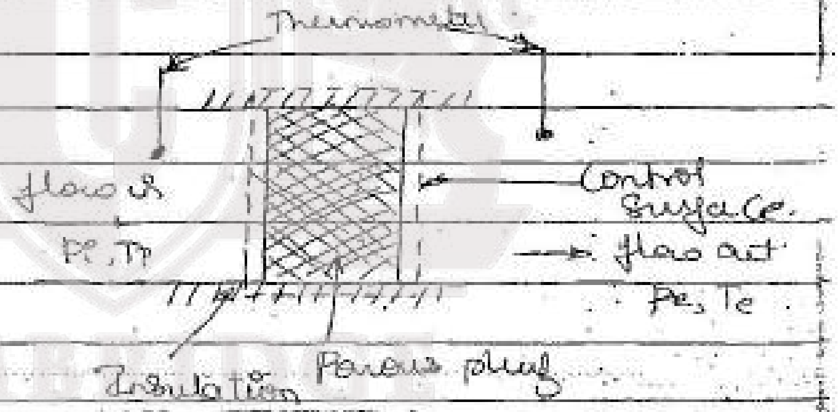
$$Q = m(h_2 - h_1)$$

Q is taken as positive because heat flows from the chamber to the evaporator.

Throttling process

Throttling process involves the passage of a higher pressure fluid through a narrow constriction. The effect is reduction in pressure and increase in volume. It is adiabatic as no heat flows from and to the system, but it is not reversible.

A porous plug experiment performed by Joule and Thomson in 1852. A stream of incompressible fluid is made to pass steadily through a porous plug placed in an insulated and horizontal pipe. The friction of the narrow passage causes the pressure to drop and exit pressure is less than intake pressure.



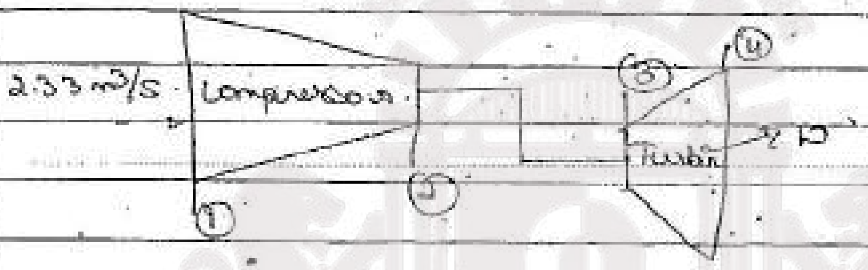
* $W_s = 0$
* $Q = 0$
* $(Z_1 = Z_2)$
* $KE = 0$

$\therefore h_1 = h_2$

Thus the throttling expansion process is an isenthalpic process.

When a real gas undergoes a throttling process a change in temperature takes place. On performing series of experiments on the same gas, keeping P_1 & T_1 constant, by varying the pressure downstream of the plug to various values P_2, P_3, P_4, after throttling T_1, T_2, T_3, T_4, etc be the corresponding temperatures. For a graph plotted for P and T, smooth curve drawn to these points will be a curve of constant enthalpy because $h_1 = h_2 = h_3 = h_4$ etc

Q. A turbocompressor delivers 2.33 m³/s at 0.276 MPa, 43°C which is heated at this pressure to 430°C & finally expanded in a turbine which delivers 1860 kW. During the expansion, there is a heat transfer of 0.09 MJ/s to the surroundings. Calculate the turbine exhaust temperature if changes in kinetic & potential energy are negligible. Assume for air $R = 0.287\,kJ/kg\,K$, $C_p = 1.005\,kJ/kg\,K$.



$P_1 = P_2 = 0.276\,MPa$ $\qquad \dot{V} = 2.33\,m^3/s \quad \therefore v = \dfrac{\dot{V}}{\dot{m}}$

$t_1 = 43°C$ $\qquad W = +1860\,kW$

$P_1 v_1 = RT_1$ $\qquad Q = -0.09\,MJ/s$

$0.276 \times 10^3 \times v_1 = 0.287 \times (273+43)$

$v_1 = 0.3286\,m^3/kg$

$\therefore$ mass flow rate $\dot{m} = \dfrac{2.33}{0.3286} = 7.09\,kg/s$

As K.E & P.E are neglected, then

SFEE is reduced to,

$\dot{m}h_3 + Q = Wh_4$

$-0.09 \times 10^3 - 1860 = 7.09(h_4 - h_3)$

$(h_3 - h_4) = 275$

(or) $t_3 - t_4 = \dfrac{275}{1.005}$ $\qquad \boxed{\dfrac{dh}{dt} = C_p}$

$t_3 - t_4 = 273.67$

$\therefore t_4 = 156.38°C \Rightarrow [430 - 273.67]$

3.b) Given :- $W = -15 kJ$, $Q = 75 kJ$

$W = +44 kJ$ $Q = -40 kJ$

② $Q = W$

$75 - 40 + x = -15 + 44$

∴ 6kJ → from the fluid is lost.

3c) $\cancel{Z_1} + h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} + \cancel{Z_2}$ $Z_1 = Z_2$ $A_1 = 0.1$

$h_1 = 3000 kJ/kg$

$h_1 = 3000$, $h_2 = 2762$, $V_1 = 60 m/s$ $h_2 = 2762 kJ/kg$

$10^3 \times 3000 + \frac{60^2}{2} = 10^3 \times 2762 + \frac{V_2^2}{2}$ $v_1 = 0.187 m^3/kg$

$v_2 = 0.498 m^3/kg$

③ $\boxed{V_2 = 692.52 m/s}$

volumetric flow $v = \frac{V}{m}$

$V = v_1 \times A = 60 \times 0.1 \Rightarrow 6 m^3/sec$ $\frac{m}{s} \times m^2$

$m = \frac{A_1 V_1}{v_1}$ $m = \frac{V}{v} = \frac{6}{0.187}$ $\bar{v} = \frac{V}{m}$

$m = 32.08 kg/s$

$v_2 = 0.498 m^3/kg$

~~$A = \frac{0.498}{692.53} = 0.023 m^3$~~

$m = \frac{AV}{v_2} \Rightarrow \frac{692.53}{0.498} =$

$A \to ?$

$A = \frac{m v_2}{V_2} = \frac{32.08 \times 0.498}{692.53}$

$A_2 = 0.023 m^2$

$\boxed{\frac{\dot{m}}{Z} = \frac{m^3}{s} \times \frac{kg}{m^3} = \frac{kg}{s}}$

D.Q.P Given: $P = V^2 + \frac{20}{V}$ $V_1 = 2m^3$, $V_2 = 6m^3$

$P_1 = 9 bar$; $P_2 = 39.33 bar$

8b) $Q = 9000 kJ$.

$W_{1-2} = \int P dV = \int V^2 + \frac{20}{V} dV$.

④ $W_{1-2} = \frac{V^3}{3} + 20 \ln V \Big|_2^6$.

$= 10^5 \left[\left(\frac{216 - 8}{3} \right) + 20 \ln \left(\frac{6}{2} \right) \right]$

~~$W_{1-2} = 7150 kJ$~~ $\boxed{W_{1-2} = 8921 kJ}$

~~$\therefore W_{1-3} = 9000 - 7150 = 1850 kJ$~~

~~$(Q = W + U)$~~

D.Q.P (3b) V-constant ($\because$ Rigid vessel)
X

$m = 10 kg$ $T_1 = 290 K$ $P_2 = 2P_1$ $R = 259.82 J/kg K$

$T_2 = 580 K$ $C_v = 0.657 J/kg^\circ K$.

change in IE $= C_v(T_2 - T_1)$.

$= 0.65 (290) = 188.5 kJ/kg$

③ change in enthalpy $= C_p (T_2 - T_1)$

——— WKT $C_p - C_v = R$

$C_p = 0.2598 + 0.65 = 0.9098 kJ/kg K$

$= 0.9098 (290)$

$= 263.84 kJ/kg$

$\therefore$ Total change in internal energy $= m (IE)$

$= 10 (188.5) = \underline{1885 kJ}$.

$\therefore$ Total change in heat transfer $= m (H.T)$.

$= 10 (263.84)$

$= \underline{2638.4 kJ}$.

c) Given : $m = 0.5\ kg/s$

$V_1 = 7\ m/s$, $P_1 = 100\ kPa$

$\bar{v}_1 = 0.95\ m^3/kg$, $V_2 = 5\ m/s$

$P_2 = 700\ kPa$, $v_2 = 0.19\ m^3/kg$

$U_2 - U_1 = 90000\ J/kg$

$Q = -58\ kW$

$W_s = ?$; $\Rightarrow \frac{d_i}{d_o} = ?$

ⓑ

$m = \frac{A_1 V_1}{v_1}$; $A_1 = \frac{0.5 \times 0.95}{7} = 0.067$

$A_1 = \frac{\pi d^2}{4} \rightarrow d = 0.292$

$m = \frac{A_2 V_2}{v_2}$; $A_2 = \frac{0.5 \times 0.19}{5} = 0.019$

$A_2 = \frac{\pi d^2}{4} \rightarrow d = 0.155$

$\frac{d_i}{d_o} = \frac{0.292}{0.155} = 0.122$

$$m\left[u_1 + P_1 v_1 + Z_1 + \frac{V_1^2}{2}\right] + Q = m\left[u_2 + P_2 v_2 + Z_2 + \frac{V_2^2}{2}\right] + W$$

$$0.5\left[P_1 V_1 + \frac{V_1^2}{2}\right] - Q = 0.5\left[(u_2 - u_1) + P_2 v_2 + \frac{V_2^2}{2}\right] + W$$

$$0.5\left[1 \times 10^5 \times 0.95 + \frac{49}{2}\right] - 58000 = 0.5\left[90000 + (7 \times 10^5 \times 0.19 + 12.5)\right] + W$$

$$\therefore W = \frac{95000}{2} + \frac{24.5}{2} - 58000 - \frac{90000}{2} - \frac{133000}{2} - \frac{12.5}{2}$$

$\therefore W = \underline{-122.004\ kJ/kg}$

Q.P(4). 3(c) ~~$P+\frac{5}{V}+$~~ $P=\left[\frac{5}{V}+1.5\right]$.

$V_1 = 0.15 m^3$.

$V_2 = 0.05 m^3$.

$Q_2 = -45 kJ$.

$_1W_2 = \text{work} = \int P dV = \int_{V_1}^{V_2}\left[\frac{5}{V}+1.5\right]dV$.

$= 5 \ln\frac{V_2}{V_1} + 1.5(V_2 - V_1)$

$= 5 \ln\frac{0.05}{0.15} + 1.5(0.05 - 0.15)$.

⑦

$_1W_2 = -5.64 kJ$

$_1Q_2 = U_2 - U_1 + {_1W_2}$

$-45 = U_2 - U_1 - 5.64$

$U_2 - U_1 = -39.352 kJ$.

$P_1 = \frac{5}{0.15} + 1.5 = 34.8 \text{ bar}$ $P_2 = \frac{5}{0.05} + 1.5 = 101.5 \text{ bar}$

$P_2V_2 - P_1V_1 = (101.5 \times 0.05) - (34.8 \times 0.15)$.

$= -0.145 kJ$.

$H_2 - H_1 = (U_2 + P_2V_2) - (U_1 - (P_1V_1))$

$= (U_2 - U_1) + (P_2V_2 - P_1V_1)$

$= -39.352 - 0.145$

$H_2 - H_1 = -39.5 kJ$.

Q.P 5) 8(b). $\frac{dW}{dT} = \frac{1}{8} kJ/°C$ & $\frac{dQ}{dT} = 0.4 kJ/°C$ $T_1 = 100°C$.

$T_2 = 260°C$.

$dW = 0.128 dT$.

⑧

$W = \int_{100}^{260} 0.128 dT \Rightarrow W = 0.128(260-100)$

$\boxed{W = 20.48 kJ}$

$dQ = 0.4dT \implies Q = \int_{100}^{260} 0.4\,dT$

$Q = 0.4(260-100) = 64\,kJ$

$\delta Q = \delta W + \Delta U_{1-2}$

$\Delta U_{1-2} = 64 - 20.48$

$\Delta U = 43.52\,kJ$

(c) $P = a + bV$; $U = 34 + 3.15PV$

$P_1 = 170\,kPa$ $\quad V_1 = 0.03\,m^3$

$P_2 = 400\,kPa$ $\quad V_2 = 0.06\,m^3$

$$WD = \int P\,dV = \int_{V_1}^{V_2} (a+bV)\,dV$$

$$= \int_{0.03}^{0.06} (a+bV)\,dV$$

ⓐ

$170 = a + 0.03b$

$400 = a + 0.06b$

$a = 170 - 230 = -60$

$\therefore b = \dfrac{230}{0.03}$

$a = 170 - 230 = -60$

$$\therefore WD = \int_{0.03}^{0.06} \left[-60 + \frac{230V}{0.03}\right] dV$$

$$-60(0.06-0.03) + \frac{230}{2 \times 0.03}(0.06^2 - 0.03^2)$$

$= -1.8 + 10.35 = 8.55\,kJ/kg$

$U_1 = 34 + 3.15 \times 170 \times 0.03 = 50.06$

$U_2 = 34 + 3.15 \times 400 \times 0.06 = 109.6$

Increase in internal Energy = 59.54 kJ

Change in enthalpy = $(P_2V_2 + U_2) - (P_1V_1 + U_1)$

$= h_2 - h_1$

$= (400 \times 0.06 + 109.6) - (170 \times 0.03 + 50.06)$

$= 32.53\,kJ$. $132.6 - 55.16 = 78.44$

$\delta Q = W + (U_2 - U_1)$

$= 8.55 + 78.44$

$= 86.99\,kJ$

P(7) (b) [illegible]

$h_1 = 2785\,kJ/kg$, $h_2 = 2512\,kJ/kg$, $V_1 = 33.3\,m/s$, $V_2 = 100\,m/s$

$Z_2 - Z_1 = 3m$, $Q = -0.29\,kJ/s$, $m = 0.42\,kg/s$

$$Q - W = m\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + (Z_2 - Z_1)g\right]$$

(b)

$$-0.29 - W = 0.42\left[(2512 - 2785) + \frac{(100^2 - 33.3^2)}{2} + \frac{9.81(3)}{1000}\right]$$

$$\boxed{W = 134.58\ kJ/s \text{ or } kW}$$

Q.P.(8) 4. d). $\gamma = 1.4$, $P_1 = 10\times10^5$ $P_2 = 1\times10^5$

$P = 12 - 5V$

$V_1 = \frac{10-12}{-5} = 0.4\,m^3/kg$.

$V_2 = \frac{1-12}{-5} = 2.2\,m^3/kg$.

⑪ $W.D = -200\,kJ/kg + \int_{V_1}^{V_2} (12-5V)\,dV$.

$W.D = \int_{0.4}^{2.2} 12(V_2-V_1) - 5\left[\frac{V_2^2 - V_1^2}{2}\right]$ $12(V_2-V_1) - 5(V_1 \cdot V$

$W.D = 12[2.2-0.4] - 5\left[\frac{2.2^2 - 0.4^2}{2}\right]$

$W.D = 21.6 - 23.4$

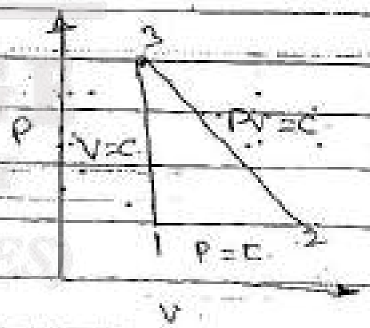
$W.D = -1.8$

Net heat transfer $= -200 - 1.8 = 198.2\,kJ$.

Q.P(9) X b)

$W_{1-2} + W_{2-3} + W_{3-1} = W_{Total}$

$P_1(V_2-V_1) + P_2V_2 \ln\frac{P_2}{P_3} + 0 =$



⑫ $V_1 = 0.028 = V_3 = 0.028\,m^3$.

$W_{1-2} = 1.4\times10^5 (V_2 - 0.028)$.

$W_{1-2} = 1.4\times10^5 V_2 - 0.0392\times10^5$

$10.5\times10^3 = 1.4\times10^5 V_2 - 0.0392\times10^5$

$V_2 = 0.103\,m^3$

$P_1 = P_2 = 1.4 \times 10^5 N/m^2$.

$W_{2-3} = 1.4 \times 10^5 \times 0.103 \ln \left[\left(\frac{0.078}{0.103}\right)\right]$.

$W_{2-3} = -18.78 kJ$.

$W_{total} = 10.5 - 18.78 + 0 = -8.28 kJ$.

$(U_2 - U_1) = C_V(T_2 - T_1) = U_2 - U_1 = +26.4 kJ/kg$

$(U_3 - U_2) = 0$.

$\therefore (U_1 - U_3) = C_V(T_1 - T_3) = -26.4 kJ$.

Heat transfer for 1-2 = $C_V(T_2 - T_1)$

$= 26.4 kJ = Q_{1-2}$.

$Q_{2-3} = 18.78 kJ$.

$Q_{3-1} = -26.4 kJ$

$Q_{net} = -8.28 kJ$

$\therefore \boxed{Q_{net} = W_{net}}$.

L.P.Q. Q. $m = 12 kg/min = 0.2 Kg/s$.

$P_1 = 1.4 \times 10^5 N/m^2$, $\rho = 25 kJ/m^3$, $V_1 = 120 m/s$.
$U_1 = 920 kJ/kg$.
⑬ $P_2 = 5.6 \times 10^5 N/m^2$, $\rho = 5 kJ/m^3$, $V_2 = 180 m/s$.
$U_2 = 720 kJ/kg$
$Z_2 = 60m$, $Q = -60 kJ/s$

$Z_1 + (P_1v_1 + u_1) + \frac{V_1^2}{2} + Q = Z_2 + (P_2v_2 + u_2) + \frac{V_2^2}{2} + W$

$v_1 = \frac{0.2}{25} = 8\times10^{-3}$

$v_2 = \frac{0.2}{5} = 0.04$

$Q + \left[1.4\times8\times10^{-3}\times10^5 + 920 + \frac{120^2}{2}\right](0.2) - 60$

$= \left[5.6\times10^5\times0.04 + 720 + \frac{180^2}{2}\right](0.2) + W$

$\boxed{W = -43.092 \text{ kJ/s}}$

P ⑩ 2 b) $u = 0.718t + 196$

$Pv = 0.287(t + 273)$

$m = 2\text{kg}, \quad P_1 = 1\text{MPa}, \quad t_1 = 100°C, \quad t_2 = 30°C$

$Q = 0, \quad W = 0$

(14)

$P_1v_1 = 0.287(100 + 273)$ $\quad$ $P_2v_2 = 0.287(30 + 273)$

$P_1v_1 = 522.45$ $\quad$ $P_2v_2 = 557.5$

$u_2 = 0.718\times30 + 196 = 217.54$

$u_1 = 0.718\times100 + 196 = 267.8$

$(P_1v_1 + u_1) + \cancel{Z_1g} + \cancel{\frac{V_1^2}{2}} + Q = (P_2v_2 + u_2) + \cancel{Z_2g} + \cancel{\frac{V_2^2}{2}} + W$

$\Delta H = (P_2v_2 + u_2) - (P_1v_1 + u_1)$

$= 825.3 - 739.99$

$\underline{\Delta H = 85.31 \text{ kJ}}$

MODULE - 2

Second law of Thermodynamics. 3rd sem BTD Rohini A
Module - 04 + 02

Devices converting heat to work (a) in a thermodynamic cycle, (b) in a mechanical cycle. Thermal reservoir $\boxed{40}$ Direct heat engine, schematic representation & efficiency. Devices converting work to heat in a thermodynamic cycle, reversed heat engine, schematic representation, coefficients of performance. Kelvin - Planck statement of the second law of Thermodynamics. PMM I & PMM II, clausius statement of second law of thermodynamics, equivalence of the two statements [Reversible & Irreversible processes, factors that make a process irreversible, reversible heat engines, carnot cycle, carnot principle] Mod 3

First law deals with conservation and conversion of energy. It states that when a thermodynamic process is carried out, energy is neither gained nor lost. Energy only transforms from one form into another and the energy balance is maintained. The law says that any change of a thermodynamic state can take place in either direction, it is not true in the inter- conversion of heat & work.

Ex: (i) The temperature of liquid contained in a vessel rises when it is stirred by paddle work. But the paddle work cannot be restored on cooling the liquid to its original state.

(2) Electric current flowing through a resistor produces heat. Electric current once dissipated as heat cannot be converted back into electricity.

(3) Fuel (coal, diesel, petrol) burns with air to form the products of combustion. Fuel once burnt cannot be restored back to its original form.

* First law fixes the exchange rate between heat & work and places no restrictions on direction of change.
* processes proceed in certain directions, but the reverse is not attainable even though the reversal of the processes does not violate the first law.
* First law does not consider the directional law which would tell whether a particular process occurs or not.

Thermal energy reservoirs or Heat reservoirs

A thermal reservoir is that part of environment which can exchange heat energy with a system. It has sufficiently large heat capacity and its temperature is not affected by the quantity of heat transferred to or from it.

The changes that take place in the thermal reservoir as heat enters or leaves are so slow and so small that processes within it are quasi-static. The reservoir which is at high temperature and supplies heat is known as heat source. ex: boiler furnace, combustion chamber and a nuclear reactor etc.

The reservoir which is at low temperature and to which heat is transferred is called the heat sink. ex: atmospheric air, ocean and river etc.

Heat Engine

A heat engine is a thermodynamic device used for continuous production of work from heat when operating in a cyclic process. Both heat and work interactions take place across the boundary of this operating device.

Characteristics of a heat engine.

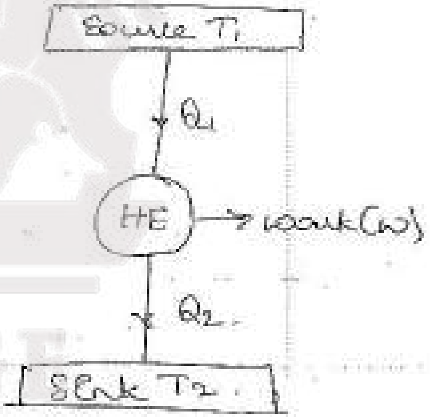
1} Heat flows from Q_1 from at a high temperature source at T_1

2} Partial conversion of heat received to mechanical work W.

3} Rejection of remaining heat Q_2 to a low temp^r sink at temp^r T_2.

4} Cyclic/continuous operation.

5} Working substance flowing through the engine.



The performance of a heat engine is given by net work output to the entire amount of heat supplied to the working medium.

Thermal efficiency is a measure of the degree of useful utilization of heat received in a heat engine.

$$\eta_{th} = \frac{\text{net work output}}{\text{total heat supplied}}$$

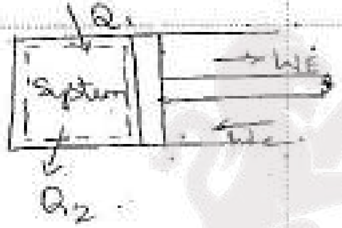
from law

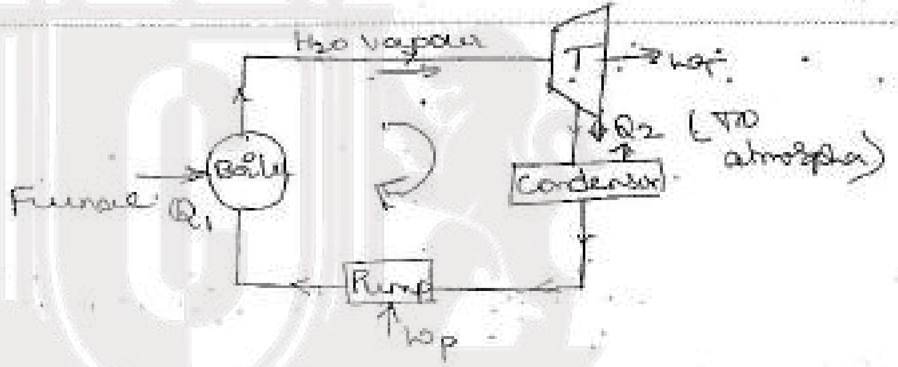
we have $\oint dW = \oint dQ$

$$W_{net} = Q_1 - Q_2$$

$$\therefore \eta_{th} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Thermal efficiency of a heat engine operating between two thermal reservoirs is always less than unity. To increase thermal efficiency Q_2 should be equal to zero, which is not possible.



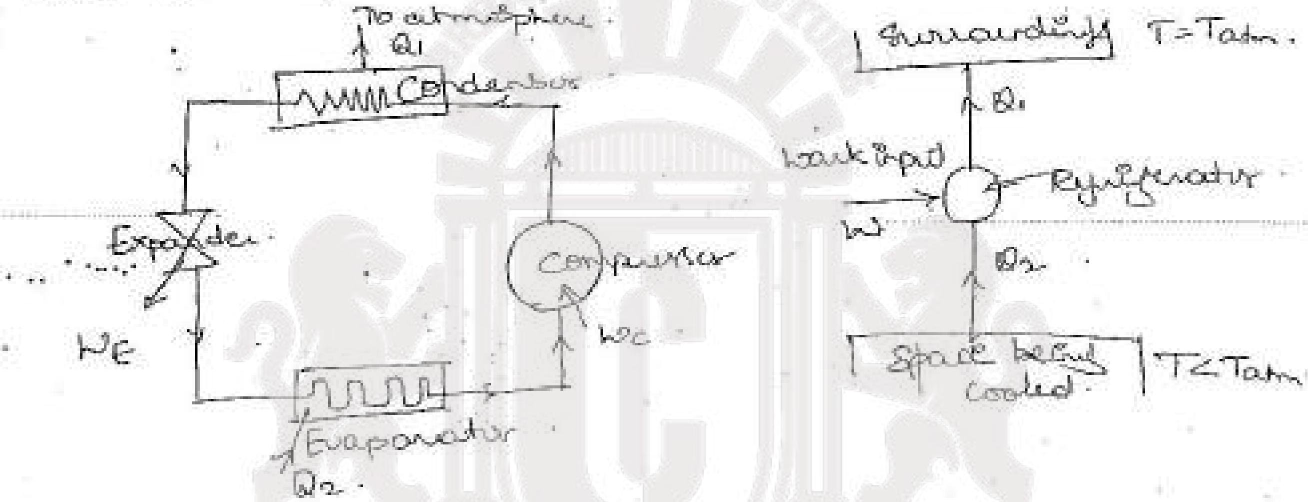


Heat Pump & Refrigerators

They are reversed heat engines. The direction of heat and work interactions are opposite to that of a heat engine i.e work input and heat output. These machines are used to remove heat from a body at low temperature level and then transfer this heat to another body at high temperature. The main purpose of the machine is to remove heat from the cooled space, it is called a refrigerator.

A refrigerator operates b/w the temp of surroundings and a tempr below that of the surroundings.

A heat pump is a device which maintains a body at a temperature higher than the temperature of the surroundings. A heat pump thus operates b/w the temp^r of the surroundings and a temp^r above that of the surroundings. Heat pumps are generally used to keep the rooms warm in winter.



COP = coefficient of Performance for an Refrigerator is defined as the ratio of desired effect to work input.

$$COP = \frac{\text{Desired effect}}{\text{work input}} = \frac{Q_2}{W_{net}}$$

$$\therefore COP = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

(or)

$$\frac{Q_1}{Q_2} - 1 = \frac{1}{COP_{ref}}$$

$$\frac{Q_1}{Q_2} = 1 + \frac{1}{COP_{ref}}$$

$$\frac{Q_1}{Q_2} = \frac{COP_{ref} + 1}{COP_{ref}}$$

$$\boxed{(\text{or})\ \frac{Q_2}{Q_1} = \frac{COP_{ref}}{1 + COP_{ref}}} \quad (i)$$

Heat pump.

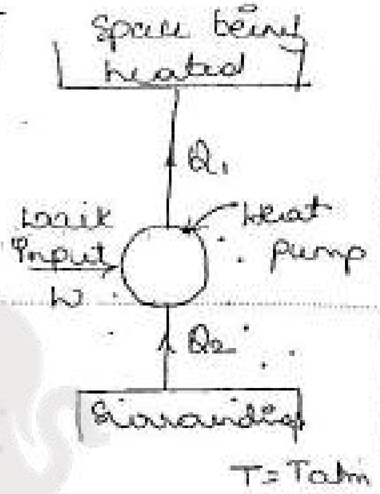
COP => For heat pump, the desired effect is the amount of heat Q_1 supplied to the space being heated.

$$COP_{HP} = \frac{Q_1}{W_{net}} = \frac{Q_1}{Q_1 - Q_2} = \frac{1}{1 - \frac{Q_2}{Q_1}}$$

$$1 - \frac{Q_2}{Q_1} = \frac{1}{COP_{HP}}$$

$$\frac{Q_2}{Q_1} = 1 - \frac{1}{COP_{HP}}$$

$$\frac{Q_2}{Q_1} = \frac{COP_{HP} - 1}{COP_{HP}} \quad \cdots\cdots \text{(ii)}$$



From (i) & (ii)

$$\frac{COP_{ref}}{1 + COP_{ref}} = \frac{COP_{HP} - 1}{COP_{HP}}$$

$$COP_{ref}\, COP_{HP} - COP_{ref} + COP_{HP} - 1 = COP_{HP} \cdot COP_{ref}$$

$$COP_{HP} = 1 + COP_{ref}$$

Thus the COP of a machine operating as a heat pump is higher than the COP of the same machine when operating as a refrigerator by unity.

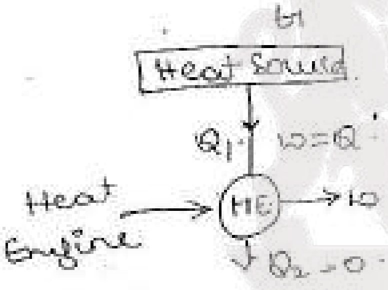
The energy conversion efficiency from energy input to desired ~~[illegible]~~ energy output taking place in a refrigerator (or) heat pump and expressed in equations. The denominator being less than the numerator, exceeds 100%, so the term coefficient of performance is used instead of efficiency.

Kelvin Planck Statement.

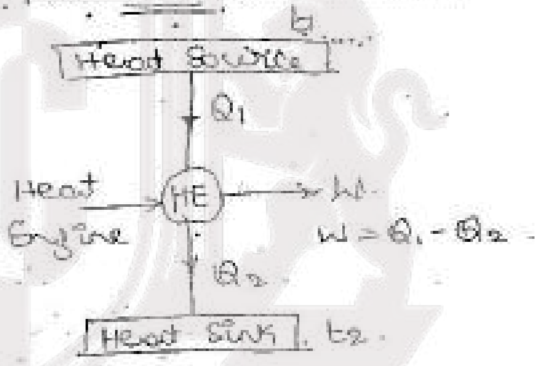
It is impossible to construct an engine that operates in a cycle and produces no effect other than work output and exchange of heat with a single heat reservoir.

The statement implies that no heat engine can be developed that receives a certain amount of heat from a high temp source and converts that into an equivalent amount of work. i.e. $W = Q$.

$$\therefore \eta_{th} = \frac{W}{Q} = 1 \text{ or } 100\%.$$



(a) Impossible. PMM2



(b) Possible.

case (a) ⇒ It satisfies the principle of energy conversion (Ist law) but violates the Kelvin statement of second law. It is called as perpetual Motion Machine 2.

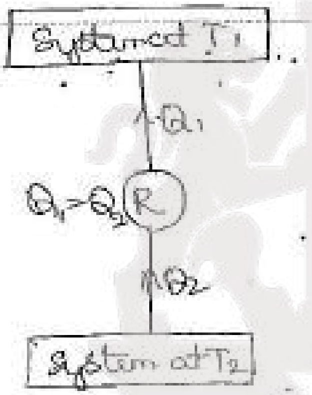
case (b) ⇒ A portion of the heat received must be rejected to a heat reservoir at low temperature. This engine receives Q_1 units of heat, rejects Q_2 units of heat and converts $(Q_1 - Q_2)$ units of heat into work per cycle.

If the bodies with which the heat engine exchanges heat are of finite heat capacities, work will be produced by the heat engine till the temperature of the two bodies are equalized.

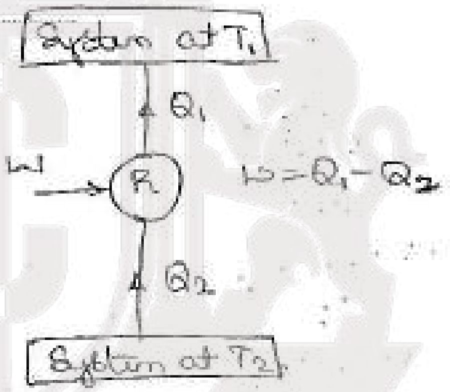
Clausius Statement

It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a system at low temperature to another system at high temperature.

The statement implies that heat cannot flow of itself from a system at low temperature to a system at high tempr.



(a) Impossible



(b) Impossible.

The COP for this arrangement will be

$$COP = \frac{Q}{W} = \frac{Q}{0} = \infty$$

Clausius statement tells that COP of a heat pump or refrigerator cannot be equal to infinity.

Both the Kelvin-Planck and Clausius statements are negative statements, they have no mathematical proof. Kelvin-Planck statement is applied to heat engines, the Clausius statement is concerned with heat pumps and refrigerators.

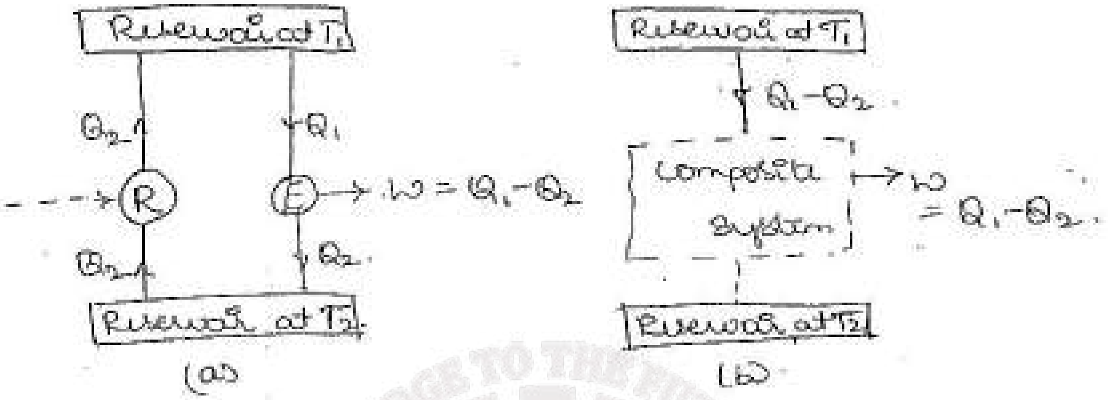
Equivalence (a) violation of clausius statement leading to violation of kelvin - Planck statement.

It is impossible to have a device satisfying one statement and violating the other. Any device that violates clausius statement leads to violation of kelvin - Planck statement and vice - versa.

Fig(a) shows a refrigerator R that operates in a cycle and transfers Q_2 units of heat from a low temperature reservoir at T_2 without any work input from an external agency [Surroundings]. This is violation of clausius statement. Along with there is a heat engine E that too operates in a cycle. This engine takes Q_1 units of heat from the high tempr reservoir, delivers $(Q_1 - Q_2)$ units of work to the surroundings & rejects the remaining Q_2 units of heat to the low temperature reservoir ; this is with Kelvin - Planck statement.

Fig (b) The heat and work interactions for the refrigerator and heat engine when coupled together. This composite system constitutes a device that receives $(Q_1 - Q_2)$ units of heat from the high temperature reservoir and converts it completely into an equivalent amount of work $W = (Q_1 - Q_2)$ without rejecting any heat to the low tempr reservoir. This operation of the composite system is in violation of the Kelvin - Planck statement.

This violation of clausius statement leads to violation of Kelvin - Planck statement also.

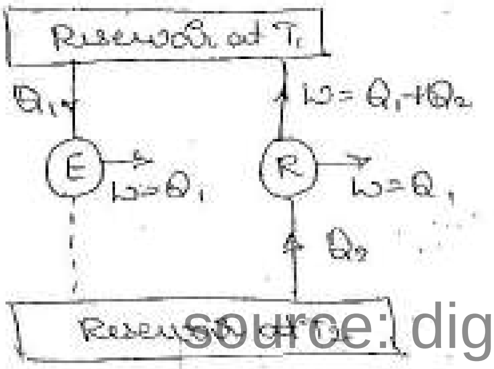


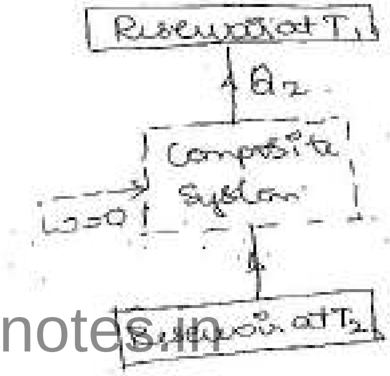
(a) (b)

Violation of Kelvin-Planck statement leading to violation of Clausius statement.

Fig (a) shows an engine E which operates from a single heat reservoir at temperature T_1. It receives Q_1 units of heat from this reservoir and converts it completely into an equivalent amount of work $W = Q_1$ without rejecting any heat to the low temp reservoir at T_2. This is in violation with the Kelvin-Planck statement. Alongwith it is a refrigerator R which extracts Q_2 units of heat from the low temperature reservoir, is supplied with Q_1 units of work from an external agency (surroundings) and supplies $(Q_1 + Q_2)$ units of heat to the high temp^r reservoir.

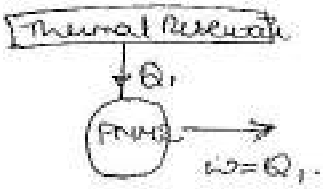
Ill'y we have a composite system fig (b).





Perpetual Motion Machine of Second Kind.

A practical heat engine exchanges heat with two thermal reservoirs source and sink and its thermal efficiency is given by



$$\eta_{th} = \frac{W_{net}}{Q_1} = \frac{Q_1-Q_2}{Q_1} = 1-\frac{Q_2}{Q_1}.$$

where Q_1 is the heat supplied from the source, Q_2 is the heat rejected to the sink & W_{net} is the net work delivered to the surroundings.

If $Q_2=0$, then $W_{net} = Q_1$ and $\eta_{th}=100\%$

If the engine exchanges heat only with one thermal reservoir, then the entire heat supplied to it gets converted into an equivalent amount of work and the efficiency becomes 100 percent. Such a heat engine is called a perpetual motion machine of the second kind, PMM2.

Module 3

Reversible and Irreversible processes

A thermodynamic process is reversible if both the system and surroundings can be restored to their respective initial states by reversing the direction of the process. A reversible process is a quasi-static process, a process carried out infinitely slowly with infinitesimal gradient with the system passing through a continuous series of equilibrium states. A reversible process results in maximum efficiency.

An irreversible process is one which is not reversible, all natural processes take place at finite speeds through finite discontinuities between the system and its environment, and is irreversible. Processes are irreversible due to factors such as friction, turbulence, diffusion, inelasticity and electrical resistance etc.

Factors that make a process irreversible.

i) Lack of equilibrium during the process.

ii) Involvement of dissipative effects

i) Lack of equilibrium during the process.

The lack of equilibrium (mechanical, thermal or chemical) b/w the system and its surroundings (or between two systems or two parts of the same system) causes a spontaneous change which is irreversible.

a) Heat transfer through a finite temperature difference

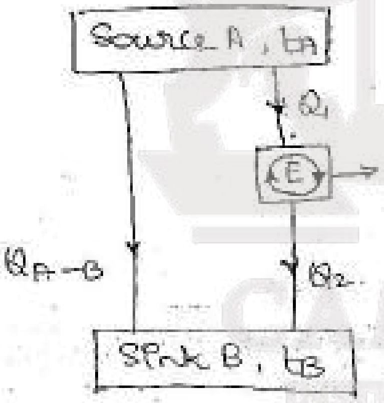
To transfer a finite amount of heat through an infinite amount of time, or infinite area, all the actual heat transfer processes are through a finite temp^r difference and are irreversible. Therefore the greater the temp^r difference, the greater is the irreversibility.

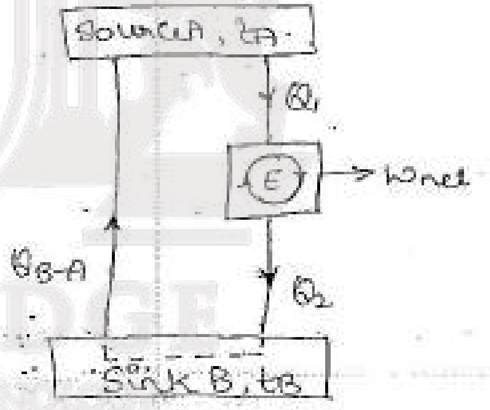
Let us assume that a source at t_A & a sink at t_B ($t_A > t_B$) are available, & let Q_{A-B} be the amount of heat flowing from A to B

Let us assume an engine operating b/w A & B, taking heat Q_1 from A and discharging heat Q_2 to B. Let the heat transfer process be reversed and Q_{B-A} be the heat flowing from B to A, and let the rate of working of the engine be such that

$$Q_2 = Q_{B-A}$$

Then the sink B may be eliminated. The net result is that E produces network W in a cycle by exchanging heat only with A, thus violating the Kelvin-Planck statement. So the heat transfer process Q_{A-B} is irreversible and Q_{B-A} is not possible.

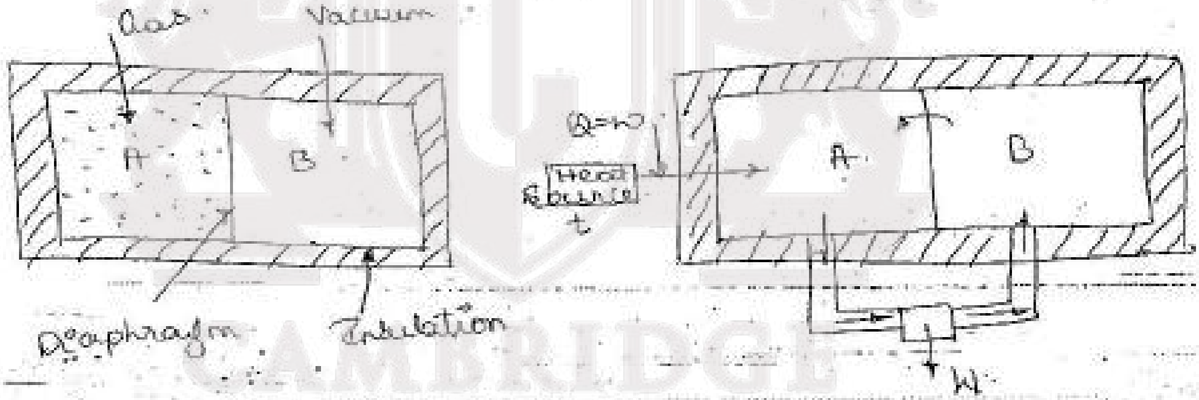




b) Free expansion

Let us consider an insulated container which is divided into two compartments A & B by a thin diaphragm. Compartment A contains a mass of gas, while compartment B is completely evacuated. If the diaphragm is punctured, the gas in A will expand into B until the pressures in A and B are equal. This is known as free or unrestrained expansion.

Let us install an engine between A & B and permit the gas to expand through the engine from A to B. The engine develops a work o/p is at the expense of the internal energy of the gas. The internal energy of the gas in B can be restored to its initial value by heat transfer Q (=W) from a source. Now, by the use of the reversed free expansion, the system can be restored to the initial state of high pressure in A and vacuum in B. We observe that the net work output is accomplished by exchanging heat with a single reservoir. This violates the Kelvin-Planck statement. Hence, free expansion is irreversible.



3) Irreversibility due to dissipative effects

The irreversibility of a process may be due to the dissipative effects in which work is done without producing an equivalent increase in the kinetic (or) potential energy of the system.

Friction, viscosity, inelasticity, electrical resistance and magnetic hysteresis. These effects are known as dissipative effects and work is said to be dissipated.

a) Friction

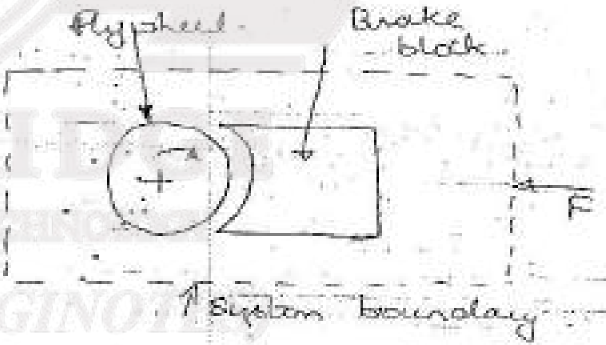
Friction is always present in moving devices. It may be reduced by suitable lubrication. The continuous motion of a movable device in the complete absence of friction is known as perpetual motion of the third kind.

Let us consider a system consisting of a flywheel and a brake block. The flywheel was rotating with a certain rpm, and it was brought to rest by applying the friction brake. The distance moved by the brake block is very small, so work transfer is very nearly equal to zero. If the braking process occurs very rapidly, there is little heat transfer. Using suffix 2 after braking and suffix 1 before braking and applying the first law, we have

$$Q_{1-2} = E_2 - E_1 + W_{1-2}$$

$$0 = E_2 - E_1 + 0$$

$$\therefore E_2 = E_1$$



The energy of the system remains constant. Since the energy may exist in the forms of kinetic, potential & molecular internal energy, we have

$$U_2 + \frac{mV_2^2}{2} + mZ_2g = U_1 + \frac{mV_1^2}{2} + mZ_1g$$

Since wheel is brought to rest, $V_2 = 0$ & there is no change in PE.

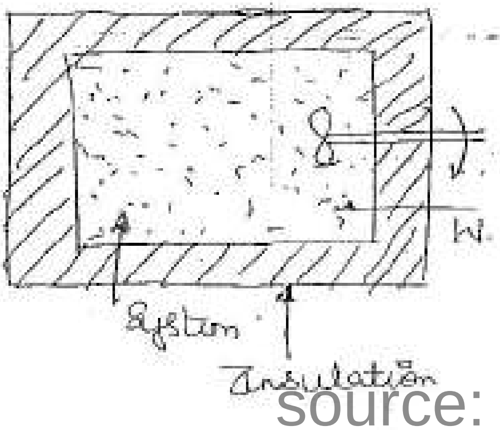
$$U_2 = U_1 + \frac{mV_1^2}{2}$$

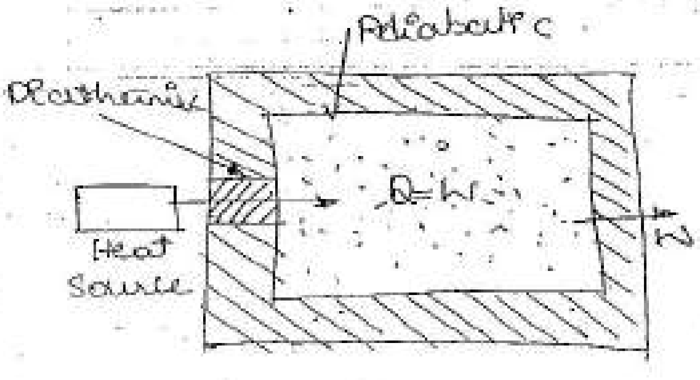
Therefore, the molecular internal energy of the system increases by the absorption of the K.E of the wheel. The reverse process, i.e the conversion of this increase in molecular internal energy into K.E within the system to cause the wheel to rotate is not possible.

iv. Paddle-wheel work transfer.

Work transfer into a system in an insulated container by means of a paddle work wheel, which is also known as stirring work.

Let us assume that the same amount of work is delivered by the system at the expense of its molecular internal energy and the temp^r of the system goes down. The system is brought back to its initial state by heat transfer from a source. These two processes together constitute a cycle in which there is work o/p & the system exchanges heat with a single reservoir. It becomes a PMM2 and hence the dissipation of stirring work to internal energy is irreversible.





Carnot cycle

A carnot cycle consists of four distinct processes, two reversible isothermal processes and two reversible adiabatic processes.

The elements needed for analysis of cycle are,

1) a working substance which is assumed to be a perfect gas.

2) two heat reservoirs. the hot reservoir at temperature T_1 and the cold reservoir at tempr T_2 (Source & Sink)

3) Piston-cylinder arrangement for getting the work out of the working substance. The piston & cylinder walls are taken as perfect heat insulators. The cylinder head is imagined to provide a diathermic cover (perfect heat conductor) and an adiabatic cover (perfect heat insulator)

The sequence of operation for the different processes constituting a carnot cycle is done assuming there is no friction to the movement of piston inside the cylinder

1. Isothermal expansion (a-b)

The heat is supplied to the working fluid at constant temp T_1. heat is supplied by bringing the heat source in good thermal contact with the cylinder head through diathermic cover. The gas expands isothermally from state point a (Pa, Va) to state point b (Pb, Vb). The heat supplied equals the work done which is represented by area under the curve a-b on pressure-volume diagram

$$\therefore\ Q_1 = W_{a-b} = P_a V_a \log_e \frac{V_b}{V_a}$$

$$= mRT_1 \log_e \frac{V_b}{V_a}$$

b) Adiabatic expansion (b-c)

At the end of isothermal expansion (point b), the heat source is replaced by adiabatic cover. The expansion continues adiabatically & reversibly upto state point C. Work is done by the working fluid at the expense of internal energy & its temperature falls to T_2 at state point c.

c) Isothermal compression (c-d)

After state point C, the piston start moving inwards & the working fluid is compressed isothermally at temp^r T_2. The constant temp^r T_2 is maintained by removing the adiabatic cover and bringing the heat sink in contact with the cylinder head. The compression continues upto state point d. The working fluid loses heat to the sink & its amount equals to work done on the working fluid. This work is represented by area under the curve c-d and its amount is given by

$$Q_2 = W_{c-d} = P_c V_c \log_e \frac{V_c}{V_d}$$

$$= mRT_2 \log_e \frac{V_c}{V_d}$$

d) Adiabatic compression (d-a).

At the end of isothermal compression, the heat sink is removed and is replaced by adiabatic cover

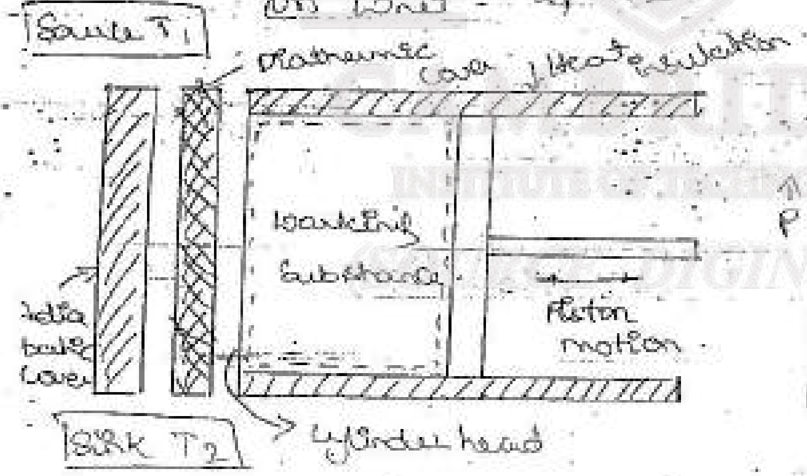
The compression now proceeds adiabatically & reversibly till the working fluid returns back to its initial state point a. Work is done on the working fluid, the internal energy increases and temp is raised to T_1.

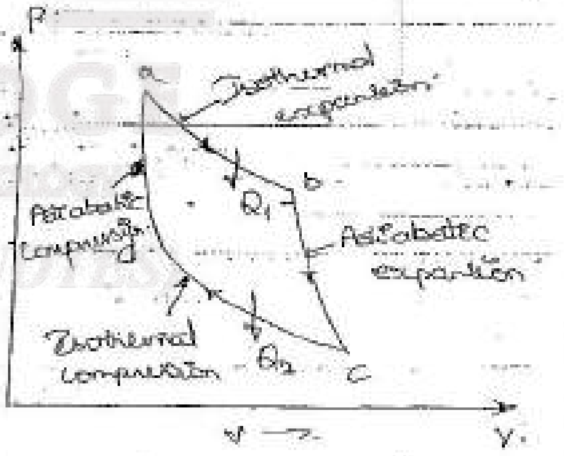
All the processes that constitute a carnot cycle are reversible, the carnot cycle is referred to as a reversible cycle. A cyclic heat engine working on the carnot cycle is called carnot engine & its thermal efficiency is given by

$$\eta = \frac{\text{net work output}}{\text{heat input}} = \frac{W_{net}}{Q_1}$$

There are no heat interactions along the reversible adiabatic processes b-c & d-a, & application of the first law of thermodynamics for the complete cycle gives $\oint \delta W = \oint \delta Q$

$$\text{(or) } W_{net} = Q_1 - Q_2$$





$$W_{net} = Q_1 - Q_2$$

$$\eta_{the} = \frac{mRT_1 \log_e \frac{V_b}{V_a} - mRT_2 \log_e \frac{V_c}{V_d}}{mRT_1 \log_e \frac{V_b}{V_a}}$$

$$\eta_{the} = 1 - \frac{T_2}{T_1} \times \log_e \frac{V_c}{V_d}$$

For the adiabatic expansion processes b-c & d-a,

$$\frac{T_b}{T_c} = \left[\frac{V_c}{V_b}\right]^{\gamma-1} \quad \& \quad \frac{T_a}{T_d} = \left[\frac{V_d}{V_a}\right]^{\gamma-1}.$$

Since $T_b = T_a = T_1$ and $T_c = T_d = T_2$,

$$\therefore \frac{T_1}{T_2} = \left(\frac{V_c}{V_b}\right)^{\gamma-1} = \left(\frac{V_d}{V_a}\right)^{\gamma-1}.$$

$$(\text{or}) \quad \frac{V_c}{V_b} = \frac{V_d}{V_a} \quad (\text{or}) \quad \frac{V_c}{V_d} = \frac{V_b}{V_a}.$$

Sub^{s} the above relation in equation, we get.

$$\eta = 1 - \frac{T_2}{T_1} = \frac{T_1 - T_2}{T_1}$$

Points to remember.

1. The efficiency is independent of the working fluid & depends upon the temperature of source & sink.

2) The efficiency is directly proportional to temperature difference $(T_1 - T_2)$ between the source and the sink.

3) Higher the temp^r difference b/w source and sink, the higher will be the efficiency obtained.

4) If $T_1 = T_2$, no work will be done & efficiency will be zero.

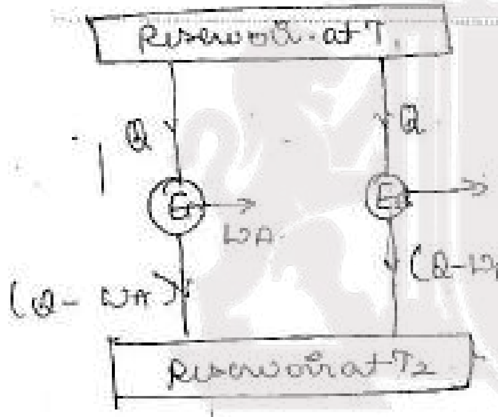
Reversed heat engine (or) Carnot heat pump

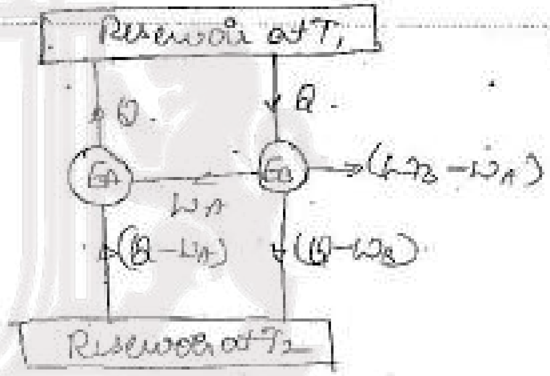
Refrigerators and heat pumps are reversed heat engines. The reversible process constituting a heat engine are individually reversed & carried out in reverse order.

when a reversible process is operated backwards, all the energy transfers associated with the process get reversed in direction, but remain same in magnitude.

carnot theorem.

No heat engine operating in a cycle between two given thermal reservoirs, with fixed temperatures, can be more efficient than a reversible engine operating b/w the same thermal reservoirs.





consider a reversible engine E_A & an irreversible engine E_B operating b/w the same thermal reservoirs at temps T_1 and T_2. For the same quantity of heat Q taken from the high temp. source, the work output from these engine is W_A & W_B respectively. As such the heat given off by the reversible engine is $(Q-W_A)$ & that from irreversible engine is $(Q-W_B)$.

Let it be presumed that the irreversible engine E_B is more efficient than the reversible engine E_A.

Then $\frac{W_B}{Q} > \frac{W_A}{Q}$ ∴ $W_B > W_A$.

& $(Q-W_A) < (Q-W_B)$. i.e. output from irreversible engine is more than that from reversible engine.

Let us have a composite system constituted by the reversible engine i.e refrigerator and the irreversible engine.

The net effect is

∴ no net interaction with the high temp^r heat reservoir

It supplies and recovers back the same amount of heat

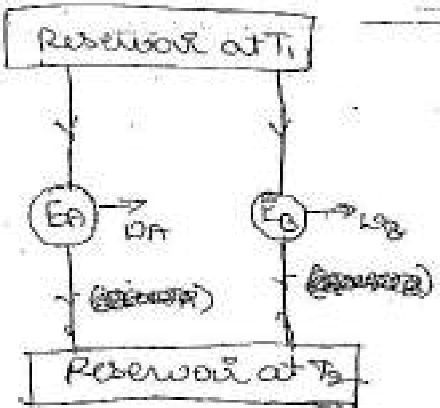
The composite system withdraws

$(Q - W_A) - (Q - W_B) = (W_B - W_A)$ units of heat from the low temp^r reservoir and converts that into equivalent amount of work output.

The combination thus constitutes a PMM of II^nd kind in violation of the second law. The assumption that the irreversible engine is more efficient than the reversible engine is wrong. Hence, an irreversible engine cannot have efficiency higher than that from a reversible engine operating b/w the same thermal reservoirs.

Corollaries of Carnot's theorem

Corollary 1: All reversible engines operating b/w the two given thermal reservoirs, with fixed temperature, hence they have same efficiency.



Let both the heat engines E_A & E_B be reversible in operation. Both operate b/w the same thermal reservoirs at temp^r T_1 & T_2 & draw the same quantity of heat Q from the high temp^r source

Let it be assumed that:

- Engine E_A is more efficient that engine E_B
- Engine E_B with lower efficiency is reversed to operate as a heat pump / refrigerator
- Both heat engines E_A and E_B can be coupled and a part of the work output of engine E_A can be used to drive the heat pump.

This assumption of making E_B as more efficient than E_A and the operating of E_A is reversed is again violating the second law.

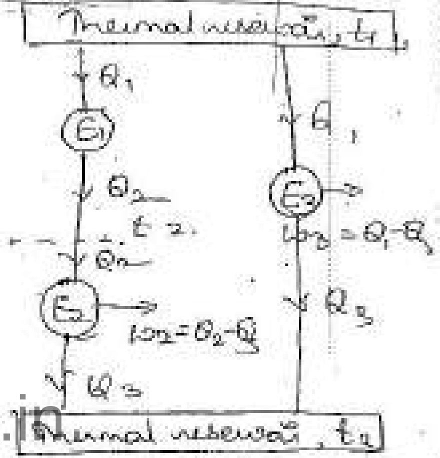
∴ Therefore, neither engine E_A nor engine E_B can be more efficient than the other. Hence all reversible engines operating between the same two thermal reservoirs must have equal efficiencies.

Thermodynamic Temperature scale.

The thermodynamic tempr scale is estab based on the fact that the efficiency of a reversible heat engine does not depend on the nature of working medium but depends only on the temperatures of the two thermal reservoirs between which it operates

$$\eta_{rev} = \phi(t_1, t_2).$$

where ϕ signifies a func of function that connects the temperature with temperature scale and is independent of the property of the working fluid



Consider two reversible heat engines E_1 & E_2 operating b/w thermal reservoirs of pairs of temp's (t_1, t_2) & (t_2, t_3). The engine E_1 receives heat from source at t_1 and rejects heat at t_2 to engine E_2 which in turn rejects heat to the sink at t_3.

∴, for the reversible engine E_1 working b/w temperature limits t_1 and t_2.

$$\eta_1 = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{1}{Q_1/Q_2} = 1 - \frac{1}{f(t_1, t_2)}$$

i.e $\frac{Q_1}{Q_2} = f(t_1, t_2)$ where f is any function.

Illy for the reversible engine E_2, working b/w t_2 & t_3

$$\frac{Q_2}{Q_3} = f(t_2, t_3)$$

The engines E_1 and E_2 constitute another heat engine E_3 operating b/w t_1 & t_3

$$\therefore \frac{Q_1}{Q_3} = f(t_1, t_3).$$

Now, we can write

$$\frac{Q_1}{Q_2} = \frac{Q_1/Q_3}{Q_2/Q_3}$$

$$f(t_1, t_2) = \frac{f(t_1, t_3)}{f(t_2, t_3)}$$

$$\frac{Q_1}{Q_2} = f(t_1, t_2).$$

The function 'f' depends upon chosen scale of temp^r and has infinite variety of forms.

If a sensible scale is selected, a tempr scale is defined which is independent of thermometric substance such a scale is called the thermodynamic tempr scale

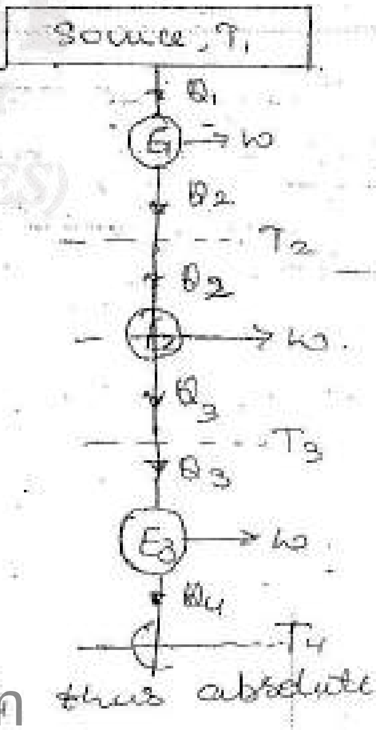
Kelvin had choosen the form of function as "T"

Thus $\frac{Q_1}{Q_2} = f(t_1, t_2) = \frac{T_1}{T_2}$

A temperature scale is which is independent of the property of thermometric substance is defined as thermodynamic temperature scale.

consider a series of reversible engines extending from a source T_1 to lower temperature such that,

- all the reversible engines operate b/w two reservoirs.
- each sink is a source for the following engine,
- Heat rejected from one engine becomes the heat supply to the next engine.
- The work done by each engine is same



The difference b/w the temperature of the successive reservoirs are the same. At one point when the heat rejected approaches zero, the tempr of heat rejection also approaches zero limit ∴ efficiency will be greater than one, the lowest possible tempr on thermodynamic scale is zero. This violates the second law of thermodynamics

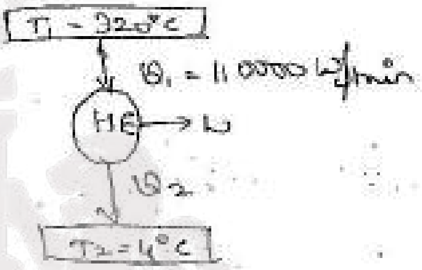
Thus heat rejected can never be zero thus absolute

[illegible]

June 2010.

1). A source X can supply energy at a rate of 11000 kJ/min at 320°c. A second source 'Y' can supply energy at a rate of 11000 kJ/min at 60°c which source 'X' or 'Y' would you choose a supply energy to an ideal reversibl engine, that is to produce a large amount of power, if the temp of surroundings is 4°C.

Source X.



$$\eta_1 = 1 - \frac{T_2}{T_1}$$

$$\eta = 1 - \frac{277}{593}$$

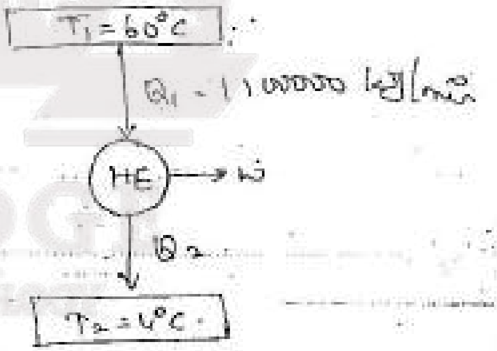
$$\underline{\eta = 53.28\%}$$

$$\eta = \frac{W}{Q_1}$$

$$W = 110000 \times 0.5328$$

$$\underline{W = 58617.21 \text{ kJ/min}}$$

Source Y



$$\eta_2 = 1 - \frac{277}{333}$$

$$\eta_2 = 0.168$$

$$\eta = 16.8\%$$

$$W = 110000 \times 0.168$$

$$\underline{W = 18498.49 \text{ kJ/min}}$$

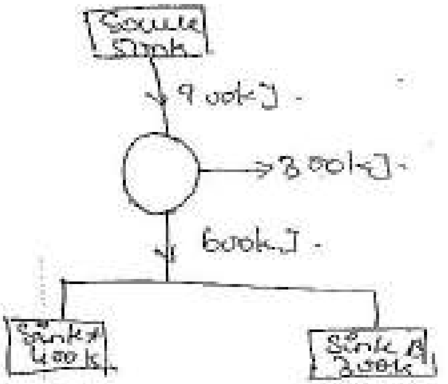
we can chose source X as the power output is large.

Dec 2010

2) A reversible heat engine is supplied 900kJ of heat from a heat source at 500K. The engine develops 300kJ of net work & rejects heat to two heat sinks at 400K & 300K. Determine the engine thermal efficiency and magnitude of heat interaction with each of sink

Sol^n.

Heat rejected = $Q_2+Q_3=600$.



Entropy.

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} + \frac{Q_3}{T_3}.$$

$$\frac{900}{500} = \frac{Q_2}{400} + \frac{Q_3}{300}$$

$$3Q_2 + 4Q_3 = 2160$$

$$Q_2 + Q_3 = 600 \,(\times 3)$$

$$Q_3 = 360\,kJ.$$

$$\therefore Q_2 = 600-360$$

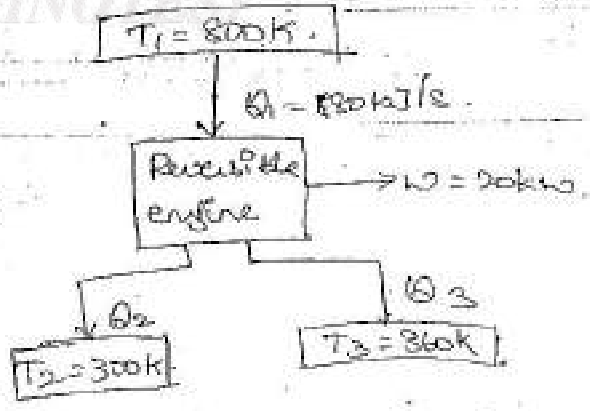
$$\boxed{Q_2 = 240\,kJ}$$

$$\eta = \frac{W}{Q_1} = \frac{300}{900} = \underline{\underline{33.33\%}}$$

3). Dec 2011

A reversible engine working in a cycle takes 4800kJ/min of heat from a source at 800K & develops 20kW power. The engine rejects heat to two reservoirs at 300K and 360K. Determine the heat rejected to each sink.

Sol^n $Q_1 = \frac{4800}{60} = 80\,kJ/s$



$$Q_1 = Q_2 + Q_3 + W.$$

$$80 = Q_2 + Q_3 + 20$$

$$Q_2 + Q_3 = 60 \longrightarrow (1)$$

Entropy balance gives

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} + \frac{Q_3}{T_3}$$

$$\frac{80}{800} = \frac{Q_2}{300} + \frac{Q_3}{360}$$

$$3.6Q_2 + 3Q_3 = 108$$

$$Q_2 + Q_3 = 60 \times 3$$

$$0.6Q_2 = 48$$

$$Q_2 = 80\,kJ$$

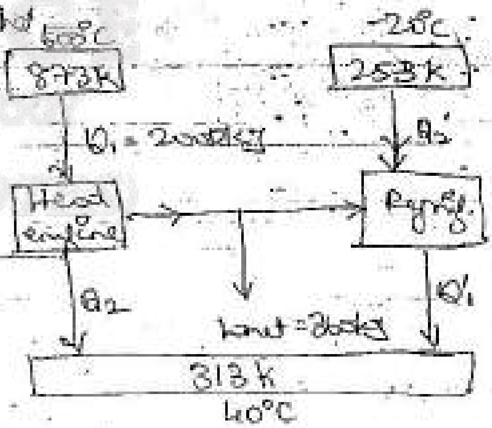
$$Q_3 = 80 - 60$$

$$\boxed{Q_3 = 20\,kJ}$$

2012.

4) A reversible heat engine operates between two reservoirs at temperatures of 600°C and 40°C. The engine drives a reversible refrigerator which operates b/w reservoirs at temperature of 40°C & -20°C. The heat transfer to the heat engine is 2000kJ and the network output of the combined engine refrigerator plant is 360kJ. Evaluate the heat transfer to the refrigerant and net heat transfer to the reservoir at 40°C

Find: Q_1', Q_2' & $Q_2 + Q_1' = Q_{net}$



$$\eta_E' = 1 - \frac{T_2}{T_1} = 1 - \frac{313}{873} = 0.64 \text{ to } 64\%$$

$$\eta_E' = \frac{W}{Q_1}$$

$$W = 2000 \times 0.6414$$

$$\boxed{W = 1282.93\,kJ}$$

$$W_{net} = W_E - W_R$$

$$360 = 1282.93 - W'$$

$$\boxed{W' = 922.93\,kJ}$$

$$COP_{ry} = \frac{253}{313-253} \quad \left[\frac{D.E}{Work}\right] = \frac{Q_2}{Q_1 - Q_2}$$

$$COP_{ry} = 4.21$$

$$COP = \frac{Q_2'}{W} \quad ; \quad 4.21 = \frac{Q_2'}{922.93}$$

$$\boxed{Q_2' = 3885.53 kJ}$$

Heat transferred to the refrigerant = $Q_2' = 3885.53 kJ$

$$Q_1' - Q_2' = W_r$$

$$Q_1' = Q_2' + W_r$$

$$Q_1' = 922.93 + 3885.53$$

$$\boxed{Q_1' = 3978.46 kJ}$$

$$Q_1 - Q_2 = W$$

$$Q_2 = Q_1 - W$$

$$Q_2 = 2000 - 1282.93$$

$$\boxed{Q_2 = 717.03 kJ}$$

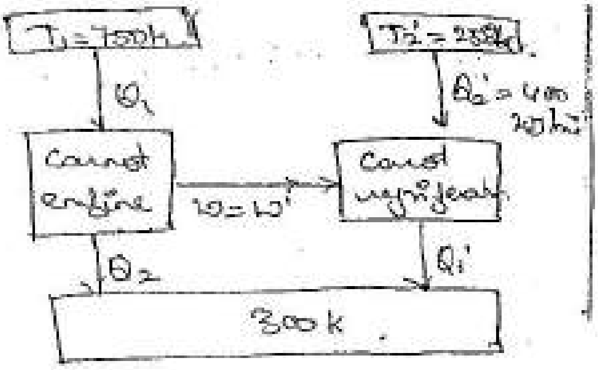
∴ Net heat transfer to 40°C reservoir = $Q_2 + Q_1'$

$$= 717.03 + 3978.46$$

$$\boxed{Q_{net} = 4695.49 kJ}$$

DEC 2012

5) A carnot engine receives heat at 750K and rejects the waste heat to the environment at 300K. The entire output of the heat engine is used to drive a carnot refrigerator that removes heat from the cooled space at −15°C at a rate of 400kJ/min and rejects to the same environment at 300K. Determine the (i) the rate of heat supplied to the heat engine (ii) the total rate of heat rejection



Soln: $\eta_{engine} = 1 - \frac{T_2}{T_1}$

$\eta = 1 - \frac{300}{750}$

$\boxed{\eta_{en} = 0.6}$

$COP_{ref} = \frac{T_2'}{T_1' - T_2'} = \frac{258}{300 - 258} = 6.1428$

$COP = \frac{Q_2'}{W'}$

$W' = \frac{400}{6.1428} = 65.1$ kJ/min

$\eta = \frac{W}{Q_1}$

$Q_1 = \frac{65.1}{0.6} = 108.52$ kJ/min

$Q_1 - Q_2 = W$

$Q_2 = Q_1 - W$

$Q_2 = 108.52 - 65.1$

$\boxed{Q_2 = 43.42 \text{ kJ/min}}$

$Q_1' - Q_2' = W'$

$Q_1' = Q_2' + W'$

$= 400 + 65.1$

$Q_1' = 465.1$ kJ/min

Total heat rejected to the environment

$= Q_2 + Q_1'$

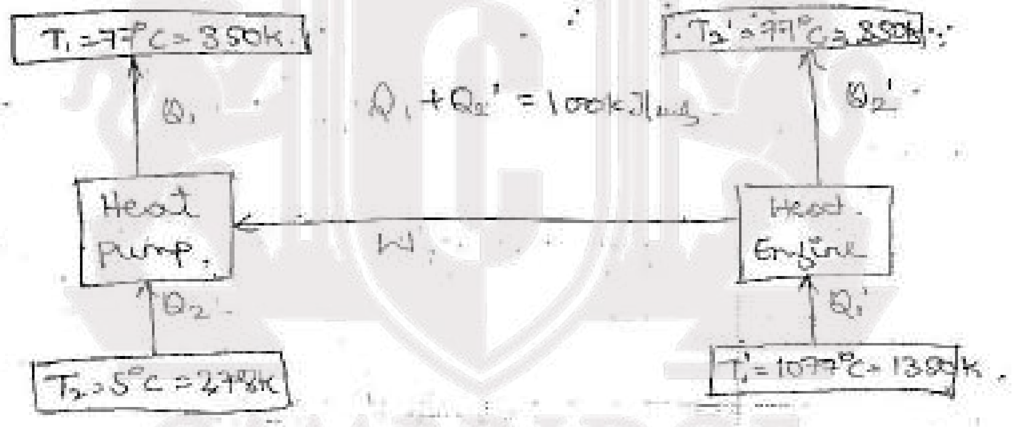
$= 43.42 + 465.1$

$\boxed{Q_{rej} = 508.52 \text{ kJ/min}}$

June 2012.

6) A heat pump working on a reversed carnot cycle takes in energy from a reservoir maintained at 5°C and delivers it to another reservoir where temperature is 77°C. The heat pump derives power for its operation from a reservoir reversible heat engine operating with in the higher and lower temperatures of 1077°C and 77°C. For every 100kJ/kg of energy supplied to reservoir at 77°C. Estimate the energy taken from the reservoir at 1077°C.

Solⁿ:



For heat pump

$$COP_{HP} = \frac{T_1}{T_1 - T_2} = \frac{350}{350-278} = 4.86.$$

$$COP = \frac{Q_1}{W} \Rightarrow W = \frac{Q_1}{COP} = \frac{Q_1}{4.86} = 0.2057 Q_1 \rightarrow (1)$$

$$\eta = 1 - \frac{T_2'}{T_1'} = 1 - \frac{350}{1350} = 0.7407.$$

$$\eta = \frac{W}{Q_1'}$$

$$W = 0.7407 Q_1' \rightarrow (2).$$

From (1) & (2).

$$0.2057Q_1 = 0.7407Q_1' \longrightarrow (3)$$

$$\frac{Q_1'}{Q_2'} = \frac{T_1'}{T_2'} = \frac{1350}{350} = 3.857.$$

$$Q_1' = 3.857Q_2' \longrightarrow (4) \quad Q_2' = \frac{Q_1'}{3.875}$$

Given $Q_1 + Q_2' = 100 \Rightarrow Q_1 = 100 - Q_2'$

$$Q_1 = 100 - \frac{Q_1'}{3.857} \longrightarrow (5).$$

from (3) & (5)

$$0.2057\left[100 - \frac{Q_1'}{3.857}\right] = 0.7407Q_1'$$

$$\boxed{Q_1' = 25.9KJ}$$

$$Q_2' = \frac{Q_1'}{3.857} = \frac{25.9}{3.857} = 6.72KJ$$

$$Q_1 = 100 - Q_2'$$

$$Q_1 = 100 - 6.72$$

$$\boxed{Q_1 = 93.28}$$

Jan 2016.

87, Using a heat engine of thermal efficiency 30% to drive a refrigerator having a COP of 5, what is the heat received by the heat engine for each MJ of heat removed from the cold body of the refrigerator.

Given: $\eta_{HE} = 30\%$.

$COP_{R} = 5$.

$Q_2' = 1MJ$.

$COP = \frac{Q_2'}{W}$

$\therefore W = \frac{Q_2'}{COP} = \frac{1}{5} = \underline{0.2MJ}$

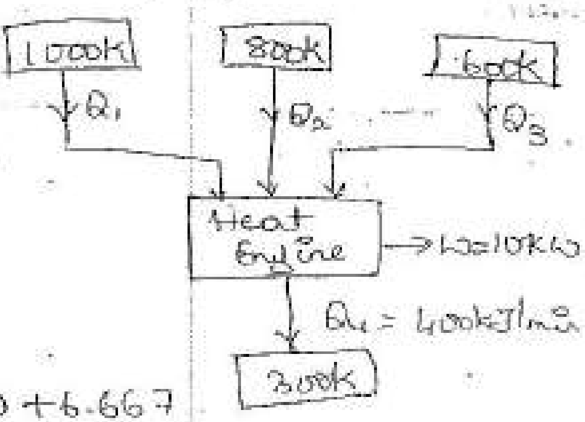
$\eta = \frac{W}{Q_1} \quad \therefore Q_1 = \frac{W}{\eta} = \frac{0.2}{0.3} = \underline{0.667MJ}$

$\boxed{Q_1 = 0.667MJ}$

July 2016

Q) A heat engine working on carnot cycle absorbs heat from three thermal reservoirs at 1000K, 800K and 600K. The engine does 10kW of network and rejects 400kJ/min of heat to a heat sink at 300K. If the heat supplied by the reservoir at 1000K is 60% of the heat supplied by the reservoir at 600K. Make calculations for the quantity of heat absorbed by each reservoir.

Soln: Heat rejected = 400kJ/min



$Q_4 = 6.667$ kJ/s or kW

$Q_1 = 0.6Q_3$

Energy balance

$Q_1 + Q_2 + Q_3 + \cancel{Q_4} = W + Q_4$

$0.6Q_3 + Q_2 + Q_3 + 6.662 = 10 + 6.667$

$\underline{Q_2 + 1.6Q_3 = 16.667}$ → (1)

Entropy balance.

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} = \frac{Q_4}{T_4}.$$

$$\frac{0.6Q_3}{1000} + \frac{Q_2}{800} + \frac{Q_3}{600} = \frac{6.667}{300}.$$

$$0.06Q_3 + \frac{Q_2}{8} + \frac{Q_3}{6} = \frac{6.667}{3}$$

$$\frac{2.88Q_3 + 6Q_2 + 8Q_3}{48} = \frac{6.667}{3}$$

$$\frac{10.88Q_3 + 6Q_2}{\cancel{48}} = 106.67 \longrightarrow (2)$$

$$Q_2 + 1.6Q_3 = 16.667 \longrightarrow (1).$$

$$6Q_2 + 10.88Q_3 = 106.67 \longrightarrow (2)$$

Solving $Q_2 = 8.33$ kW

$Q_3 = 5.209$ kW

$Q_1 = 3.125$ kW

Module 3

Reversibility: Defn of a reversible process, reversible heat engine. Importance & superiority of a reversible heat engine & irreversible process, factors that make a process irreversible, reversible heat engine, unrestricted expansion, remarks on Carnot's engine, internal & external reversibility, definition of the thermodynamic tempr scale.

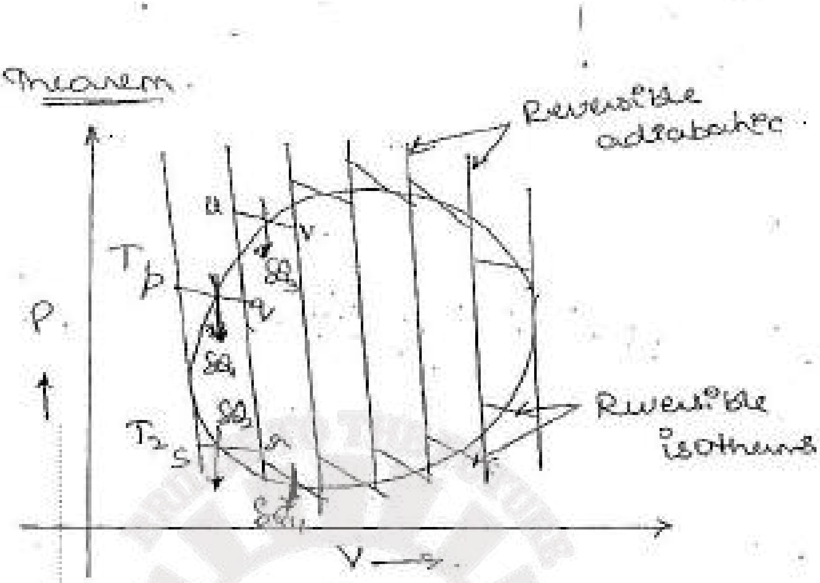
Entropy: Clausius inequality, statement - proof, entropy - definition, a property, change of entropy, entropy as a quantitative test for irreversibility, principle of increase of entropy, calculation of entropy using TdS relations, entropy as a coordinate.

Entropy

Entropy is an extensive property as it depends on the mass of the system. The unit of entropy is kJ/K while that of specific entropy is kJ/kg K. The entropy of a system increases with absorption of heat and decreases with heat rejection. For a reversible adiabatic process $\delta Q = 0$ and so will be the entropy change.

Thus Entropy is a property of the system that remains constant during a reversible adiabatic process. The reversible adiabatic process during which entropy remains constant is called isentropic process. Entropy is a measure of the degree of randomness of the molecules comprising the system. Higher the disorder greater is the increase in entropy.

Clausius Theorem.



consider a smooth closed curve which represents a reversible cycle. A number of reversible adiabatic can be drawn. The closed cycle can be split into a large number of small strips. Each of the strips are closed by drawing reversible isotherms at top as well as at the bottom. Therefore the original cycle is replaced by a large number of carnot's cycles. Now the cycles having alternate adiabatics & isotherms.

For the elemental cycle PQRS, δQ_1 is heat absorbed reversibly at T_1 and δQ_2 is the heat rejected reversibly at T_2.

$$\frac{\delta Q_1}{T_1} = \frac{\delta Q_2}{T_2} \longrightarrow (1)$$

Heat rejected is negative, we have.

$$\frac{\delta Q_1}{T_1} + \frac{\delta Q_2}{T_2} = 0 \longrightarrow (2)$$

|||ly for every other cycles,

$$\frac{\delta Q_3}{T_3} + \frac{\delta Q_4}{T_4} = 0 \longrightarrow (3)$$

∴ The whole original cycle is

$$\frac{\delta Q_1}{T_1} + \frac{\delta Q_2}{T_2} + \frac{\delta Q_3}{T_3} + \frac{\delta Q_4}{T_4} + \cdots = 0$$

$$(\text{or}) \oint_R \frac{\delta Q}{T} = 0 \longrightarrow (4)$$

Thus for a reversible cycle the cyclic integral of $\frac{\delta Q}{T}$ is equal to zero. This is known as clausius theorem. The letter R in eqn (4) indicates that this equation is valid only for reversible cycle.

Entropy is a point function

Consider a reversible cycle 1-A-2-B-1, from clausius theorem we have

$$\oint \frac{\delta Q}{T} = 0 \Rightarrow \int_{1A}^{2A} \frac{\delta Q}{T} + \int_{2B}^{1B} \frac{\delta Q}{T} \longrightarrow (1)$$

1-A-2-C-1

$$\oint \frac{\delta Q}{T} = 0 = \int_{1A}^{2A} \frac{\delta Q}{T} + \int_{2C}^{1C} \frac{\delta Q}{T} \longrightarrow (2)$$

Substracting eqn (1) from eqn (2), we have

$$\cancel{\int_{1A}^{2A} \frac{\delta Q}{T}} + \int_{2C}^{1C} \frac{\delta Q}{T} - \cancel{\int_{1A}^{2A} \frac{\delta Q}{T}} - \int_{2B}^{1B} \frac{\delta Q}{T} = 0$$

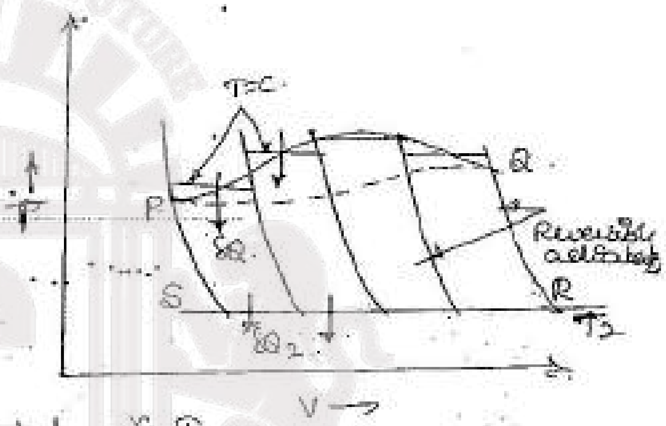
$$\int_{2C}^{1C} \frac{\delta Q}{T} = \int_{2B}^{1B} \frac{\delta Q}{T}$$

This quantity $\frac{\delta Q}{T}$ is same for paths B & C, & hence it does not depend on the path followed, but depends only on end states. Therefore it is a point function and represents a property. This property is called entropy.

Clausius Inequality.

Clausius inequality states that $\oint \frac{\delta Q}{T} \leq 0$.

Consider a cycle PQRS where process PQ is a general process either reversible or irreversible, whereas the other processes in the cycle are reversible.



Let the cycle be divided into a number of elementary cycles. For one such elementary cycle, we have

$$\eta = 1 - \frac{\delta Q_2}{\delta Q}$$

where, dQ = Heat supplied at temp^r T

dQ$_2$ = Heat rejected at temp^r T$_2$

It is known that the efficiency of a general cycle will be equal to or less than the efficiency of reversible cycle.

$$\therefore 1 - \frac{\delta Q_2}{\delta Q} \leq \left(1 - \frac{\delta Q_2}{\delta Q}\right)_{rev} \longrightarrow (1)$$

$$\frac{\delta Q_2}{\delta Q} \geq \left(\frac{\delta Q_2}{\delta Q}\right)_{rev}$$

$$\text{or } \frac{\delta Q}{\delta Q_2} \leq \left(\frac{\delta Q}{\delta Q_2}\right)_{rev} \longrightarrow (2)$$

But from Carnot's third theorem.

$$\left(\frac{\delta Q}{\delta Q_2}\right)_{rev} = \frac{T}{T_2} \longrightarrow (3)$$

from (2) & (3)

$$\text{or } \frac{\delta Q}{\delta Q_2} \leq \frac{T}{T_2}$$

(or) $\frac{\delta Q}{T} \le \frac{\delta Q_2}{T_2} \longrightarrow (4).$

The above equation holds good for any process PQ, either reversible or irreversible process.

But for a reversible process

$$ds = \frac{\delta Q_{rev}}{T} = \frac{\delta Q_2}{T_2} \longrightarrow (5).$$

Hence for any process PQ from (4) & (5).

$$\frac{\delta Q}{T} \le ds.$$

For a cycle.

$$\oint \frac{\delta Q}{T} \le \oint ds$$

But w.k.T entropy is a property and hence cyclic integral of a property is zero

$$\oint \frac{\delta Q}{T} \le 0$$

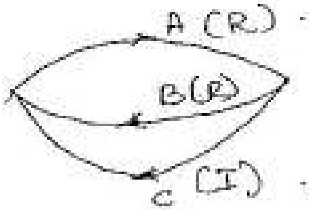
This is clausius inequality.

$\oint \frac{\delta Q}{T} = 0$, the cycle is reversible.

$\oint \frac{\delta Q}{T} < 0$, the cycle is irreversible.

$\oint \frac{\delta Q}{T} > 0$, the cycle is impossible.

change in entropy in an irreversible process



For the reversible cycle 1-A-2-B-1

$$\oint \frac{\delta Q}{T}=0 \qquad \int_{1A}^{2A}\frac{\delta Q}{T}+\int_{2B}^{1B}\frac{\delta Q}{T}=0 \longrightarrow (1)$$

For the irreversible cycle 1-A-2-C-1

$$\oint \frac{\delta Q}{T}\le 0 \qquad \int_{1A}^{2A}\frac{\delta Q}{T}+\int_{2C}^{1C}\frac{\delta Q}{T}\le 0 \longrightarrow (2)$$

(1) - (2) gives

$$\int_{1A}^{2A}\cancel{\frac{\delta Q}{T}}+\int_{2B}^{1B}\frac{\delta Q}{T}-\int_{1A}^{2A}\cancel{\frac{\delta Q}{T}}-\int_{2C}^{1C}\frac{\delta Q}{T}\ge 0 \longrightarrow (3)$$

$$\int_{2B}^{1B}\frac{\delta Q}{T}-\int_{2C}^{1C}\frac{\delta Q}{T}\ge 0 \longrightarrow (4)$$

But path B is reversible

$$\int_{2B}^{1B}\frac{\delta Q}{T}=\int_{2B}^{1B}dS \longrightarrow (5)$$

From (4) & (5)

$$\int_{2B}^{1B}dS-\int_{2C}^{1C}\frac{\delta Q}{T}\ge 0 \longrightarrow \cancel{(6)}$$

But entropy is a point function.

$$\therefore \int_{2B}^{1B} dS = \int_{2C}^{1C} dS \longrightarrow (7)$$

∴ from (6) & (7)

$$\int_{2C}^{1C} dS - \int_{2C}^{1C} \frac{\delta Q}{T} \geq 0$$

(or) $dS \geq \frac{\delta Q}{T}$

∴ The effect of irreversibility is to increase the entropy of the system.

$$\therefore dS = \frac{\delta Q}{T} + I$$

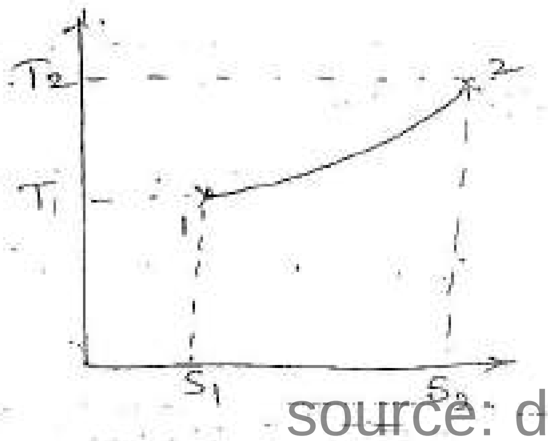
where I is called the irreversibility

Two equations of Entropy

$TdS = du + pdv$

$TdS = dh - vdp$

i) constant volume process



Considering for unit mass

$\delta Q = \delta w + du$

$\delta Q = C_v\, dT$

$\because \overset{C_v}{\cancel{C_p}} = \frac{du}{dT}$

÷ by T

$$\frac{\delta Q}{T} = C_v \frac{dT}{T}$$

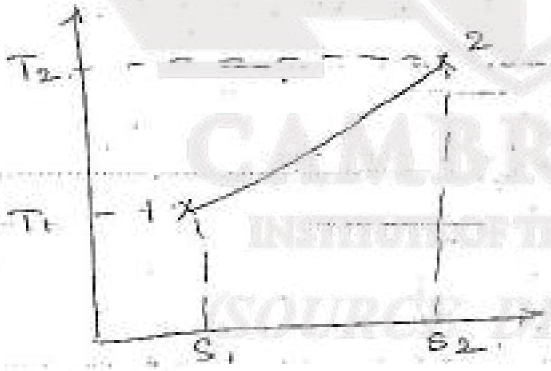
(or) $ds = C_v \frac{dT}{T}$

Integrating on b.s. we obtain

$$\int_1^2 ds = C_v \int_1^2 \frac{dT}{T}$$

$$\therefore \boxed{S_2 - S_1 = C_v \log_e \frac{T_2}{T_1} = C_v \log_e \frac{P_2}{P_1}}$$

* Constant pressure process



$\delta Q = C_p\, dT$

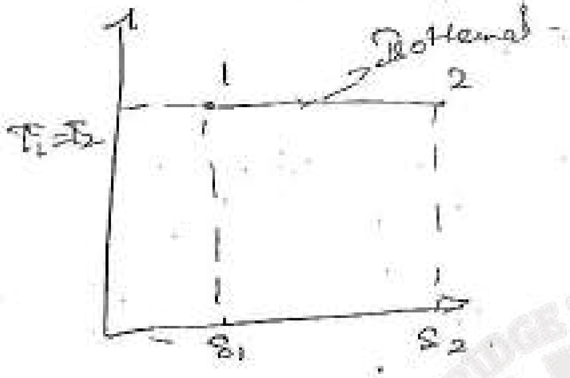
$$\frac{\delta Q}{T} = C_p \frac{dT}{T}$$

$$ds = C_p \frac{dT}{T}$$

(or) $\int_1^2 ds = C_p \int_1^2 \frac{dT}{T}$

$$\therefore \boxed{S_2 - S_1 = C_p \log_e \frac{T_2}{T_1} \text{ (or) } C_p \log_e \frac{V_2}{V_1}}$$

3) Isothermal process ,



For unit mass of substance.

$$\delta q = du + pdv$$

since the temperature remains constant, $du = 0$. &

$$\delta q = pdv$$

(or) $\frac{\delta q}{T} = \frac{P}{T} dv = R\frac{dv}{v}$

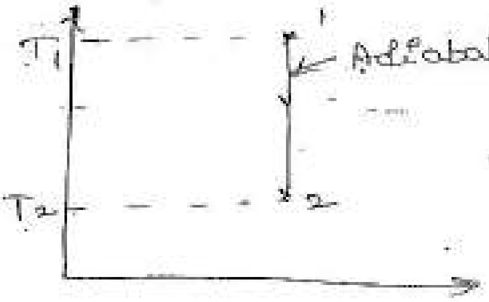
(or) $ds = R\frac{dv}{v}$

Integrating b.s , we get

$$\int_1^2 ds = R\int_1^2 \frac{dv}{v}$$

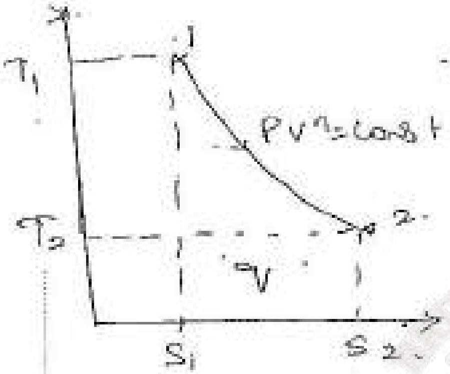
$\therefore S_2 - S_1 = R\log_e \frac{v_2}{v_1} = R\log_e \frac{P_1}{P_2}$

4) Adiabatic (Isentropic)



(or) $\frac{\delta Q}{T} = 0$ ∴ $ds = 0$.

⊛ Polytropic process -



$$\delta q = du + pdv$$

$$\delta q = C_v dT + pdv$$

$$\text{so } \frac{\delta q}{T} = C_v \frac{dT}{T} + \frac{P}{T} dv$$

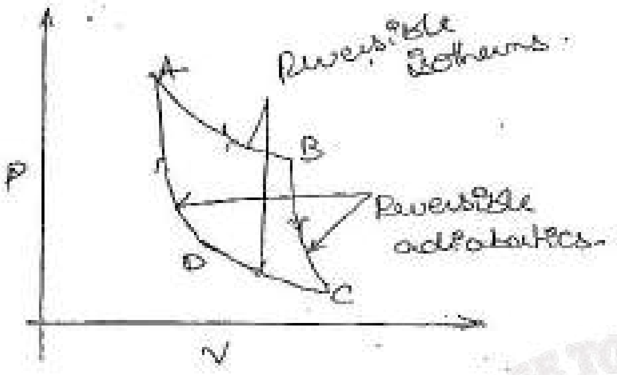
$$\text{or } ds = C_v \frac{dT}{T} + R \frac{dv}{v}$$

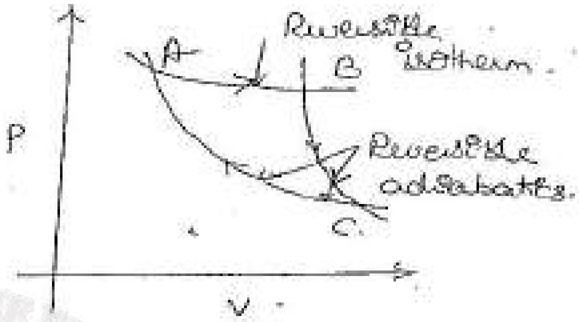
$$\int_1^2 ds = C_v \int_1^2 \frac{dT}{T} + R \int \frac{dv}{v}$$

$$\boxed{\therefore S_2 - S_1 = C_v \log_e \frac{T_2}{T_1} + R \log_e \frac{v_2}{v_1}}$$

Two reversible adiabatic paths cannot intersect each other.

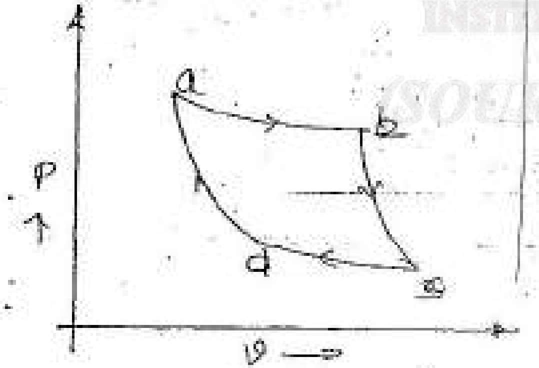
Consider a carnot cycle which consists of two reversible isotherms and two reversible adiabatics, now let two adiabatics intersect each other i.e AC & BC. A reversible isotherm is drawn so that it intersects the reversible adiabatics at A and B. Now we have a reversible cycle ABC and the included area ABC represents the net work output of the cycle.

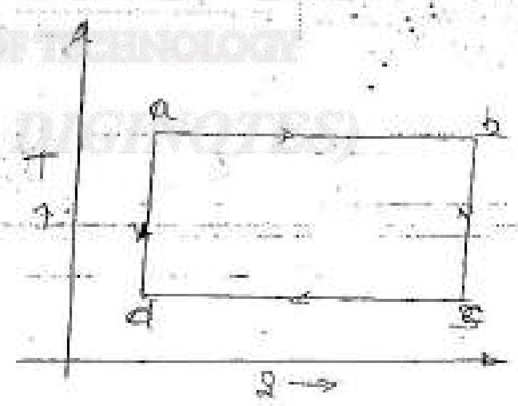




When this cycle is compared with the carnot cycle, we can see that, heat rejected is zero. Then we have this cycle ABC which is producing work by exchanging heat with a single reservoir during the process AB, which violates the kelvin Planck statement of the second law of thermodynamics. Thus the assumption that the reversible adiabatics intersect each other is wrong. Therefore the reversible adiabatic paths cannot intersect each other.

Carnot cycle on temperature - entropy diagram





A carnot cycle consists of two reversible isothermals and two reversible adiabatics. Accordingly a carnot cycle would plot as a rectangle on T-S diagram.

Process a-b : Isothermal expansion at const. temp^r T_1 during which heat Q_1 is supplied to the system.

Q_1 = area under the process line a-b

$= T_1 dS$.

Process b-c : Adiabatic expansion during which work is done by the system.

Process c-d : Isothermal compression at const. temperature T_2 during which heat Q_2 is rejected by the system.

Q_2 = area under the process line c-d.

$Q_2 = T_2 dS$

Process d-a : Adiabatic compression during which work is done on the system.

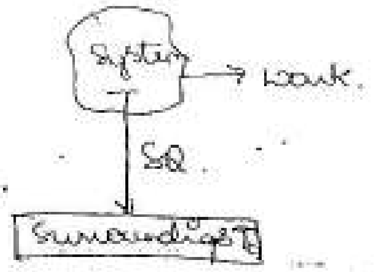
$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 dS - T_2 dS}{T_1 dS} = \frac{T_1 - T_2}{T_1}$$

The area of the rectangle abcd represents the work output per cycle and it equals,

$$Q_1 - Q_2 = (T_1 - T_2) dS$$

Principle of increase of Entropy :

Consider a system at temp^r T doing some work and transferring heat δQ to the surroundings at temperature T_0.



For the system we can write,

$$dS_{sys} \geq \frac{-\delta Q}{T}$$

For the surroundings at temp^r T_0.

$$dS_{sur} \geq \frac{\delta Q}{T_0}$$

$$dS_{sys} + dS_{sur} \geq \delta Q \left[-\frac{1}{T} + \frac{1}{T_0}\right]$$

But T is always greater than T_0.

$$\therefore \left[-\frac{1}{T} + \frac{1}{T_0}\right] > 0 .$$

$$\therefore dS_{sys} + dS_{sur} > 0 .$$

This is called the principle of increase of entropy

This equation states that the entropy of the universe is continually increasing

Tds equations of entropy: (Derivation in Ideal gases chapter).

Problems: 2010

1. one kg of ice at $-5°C$ is exposed to the atmosphere which is at $20°C$. The ice melts and comes into thermal equilibrium with atmosphere. (i) Determine the entropy increase of the universe) and (ii) what is the minimum amount of work necessary to convert the water back into ice at $-5°C$. Take $C_p = 2.093\,kJ/kg°C$ & latent heat of fusion of ice $= 333.3\,kJ/kg$.

Heat absorbed by ice from the atmosphere

= Heat absorbed in solid phase + latent heat + heat absorbed in liquid phase

$$= 1 \times C_{pi}\left[T_{if} - T_{i}\right] + L.H + 1 \times C_{pw}\left(T_{wf} - T_{wi}\right)$$

$$= 1 \times 2.093\left[0-(-5)\right] + 333.3 + 1 \times 4.187\,(20-0)$$

$Q_{ice} = 427.5\,kJ$

∴ Entropy change in atmosphere

$$(\Delta S)_{atm} = \frac{-Q}{T} = \frac{-427.5}{293} = -1.46\,kJ/K$$

Entropy change of the ice as it gets heated from $-5°C$ to $0°C$

$$(\Delta S)_1 = \int_{268}^{273} mC_p\frac{dT}{T} = 1 \times 2.093 \ln\frac{273}{268}$$

$$\underline{\underline{(\Delta S)_1 = 0.0389\,kJ/K}}$$

Entropy change of ice as it melts at 0°C to become water at 0°C.

$$(\Delta S)_2 = \frac{\text{Latent heat}}{\text{temp}^r} = \frac{333.3}{293} = \underline{1.22\ kJ/K}$$

Entropy change of water as it gets heated from 0°C to 20°C.

$$(\Delta S)_3 = \int_{273}^{293} mC_p \frac{dT}{T} = 1 \times 4.187 \ln\frac{293}{273} = \underline{0.296\ kJ/K}.$$

∴ total change in entropy of ice as it melts in to water.

$$(\Delta S)_{sys} = 0.0389 + 1.22 + 0.296$$

$$\underline{(\Delta S)_{sys} = 1.5549\ kJ/K}$$

∴ Entropy increase of universe.

$$(\Delta S)_{univ} = (\Delta S)_{sys} + (\Delta S)_{atm}$$

$$(\Delta S)_{univ} = 1.5549 - 1.46$$

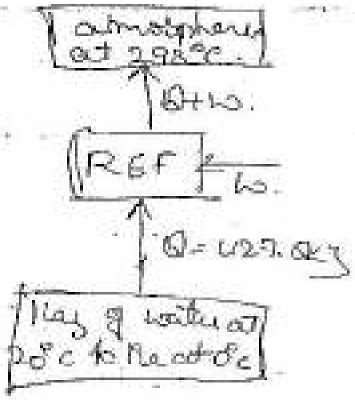
$$\underline{(\Delta S)_{univ} = 0.0949\ kJ/K}$$

(ii) To convert 1kg of water at 20°C to ice at -5°C, 427.5kJ of heat will have to be removed from it.

$$(\Delta S)_{ref} = 0$$

The entropy change of the atmosphere

$$(\Delta S)_{atm} = \frac{Q+W}{T}$$



Entropy change of the universe.

$$(\Delta S)_{univ} = (\Delta S)_{sys} + (\Delta S)_{ref} + (\Delta S)_{atm}$$

$$= -1.5549 + \frac{Q+W}{T} \quad (\Delta S_{sys} \text{ is negative})$$

By the principle of entropy.

$(\Delta S)_{univ}$ or isolated system ≥ 0.

$$-1.5549 + \frac{Q+W}{T} \geq 0.$$

$$\frac{Q+W}{T} \geq 1.5549$$

$$W_{min} = 1.5549 \times T - Q.$$

$$= 12.5549 \times 293 - 427.5$$

$$\boxed{W_{min} = 28.5 kJ}$$

July 2011.

Q. A heat engine is supplied with 278 kJ/s of heat at a constant fixed tempr of 283°C and the heat rejection takes place at 5°C. The following results were received.

(i) 208 kJ/s of heat rejected

(ii) 139 kJ/s of heat rejected

(iii) 70 kJ/s of heat rejected.

Classify which of the results report a reversible cycle, irreversible cycle or impossible cycle

(i). when $Q_2 = 208$ kJ/s.

$$\oint \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

$$= \frac{278}{556} - \frac{208}{278}$$

$$\boxed{\oint \frac{\delta Q}{T} = -0.248}$$

∴ cycle is irreversible

$$\underline{\Delta S < 0}$$

(ii) $Q_2 = 139$ kJ/s.

$$\oint \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

$$= \frac{278}{556} - \frac{139}{278}$$

$$\boxed{\oint \frac{\delta Q}{T} = 0}$$ ∴ cycle is reversible.

(iii) $Q_2 = 70$ kJ/s.

$$\oint \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

$$= \frac{278}{556} - \frac{70}{278}$$

$$\boxed{\oint \frac{\delta Q}{T} = 0.248}$$ >0 ∴ cycle is impossible.

2011

8) A heat engine absorbs 200 kJ/sec of heat at 227°C & rejects heat at 27°C. Three separate case of heat rejection are reported

i) 180 kJ/s heat is rejected

ii) 120 kJ/s heat is rejected

iii) 60 kJ/s heat is rejected. Classify each cycle

i) $Q_2 = 180$ kJ/s

$$\oint \frac{\delta Q}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

$$= \frac{200}{500} - \frac{180}{300}$$

$$\boxed{\oint \frac{\delta Q}{T} = -0.2}$$ <0 ∴ The cycle is irreversible.

ii) $Q_2 = 120 kJ/s$

$$\oint \frac{\delta Q}{T} = \frac{200}{500} - \frac{120}{300}$$

$\boxed{\oint \frac{\delta Q}{T} = 0}$ ∴ The cycle is reversible.

iii) $Q_3 = 60 kJ/s$.

$$\oint \frac{\delta Q}{T} = \frac{200}{500} - \frac{60}{300}$$

$\boxed{\oint \frac{\delta Q}{T} = 0.2}$ $\underline{>0}$ The cycle is impossible

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June 2012.

Q. 2kg of water at 80°C are mixed adiabatically with 3kg of water at 30°C in a constant pressure process at 1 atm. Determine the increase in entropy due to the mixing process. Assume for water $C_p = 4.187 kJ/kg$

Given: $m_1 = 2kg$ $\quad m_2 = 3kg$

$T_1 = 80+273 = 353K$ $\quad T_2 = 30+273 = 303K$

$C_p = 4.187 kJ/kg$

Energy lost by 2kg water = Energy gained by 3kg water

$$m_1 C_p (T_1 - T) = m_2 C_p (T - 303)$$

$$2(353 - T) = 3(T - 303)$$

$$\boxed{T = 323 K}$$

$$\Delta S_1 = \int_{T_1}^{T} m C_p \frac{dT}{T} = m C_p \ln \frac{T}{T_1}$$

$$\Delta S_1 = 2 \times 4.187 \ln\left(\frac{323}{353}\right)$$

$\Delta S_1 = -1.159 kJ/kg K$

$$\Delta S_2 = \int_{T_2}^{T} mc_p \frac{dT}{T} = mc_p \ln \frac{T}{T_2}.$$

$$\Delta S_2 = 3 \times 4.187 \ln \frac{323}{303}.$$

$$\Delta S_2 = 0.80289 \text{ kJ/kg K}.$$

Total change in entropy

$$\Delta S = \Delta S_1 + \Delta S_2.$$

$$= -1.1156 + 0.80289.$$

$$\boxed{\Delta S = -0.3127 \text{ kJ/kg.K}}.$$

5) 1.5kg of air initially at 25°C is heated reversibly at constant pressure until volume is doubled and heated reversibly until pressure is doubled at constant volume. For the total path, determine (i) the work transfer (ii) the heat transfer and (iii) the change in entropy.

(i) constant Pressure process (1-2).

$$V_2 = 2V_1$$

$$P_1V_1 = mRT_1$$

$$= 1.5 \times 0.287 \times 298$$

$$= 128.289 \text{ kJ}.$$

$$W.D = \int P dV = P\int dV = P_1(V_2 - V_1) = P_1(2V_1 - V_1).$$

$$W.D = P_1V_1$$

$$\boxed{W.D. = 128.289 \text{ kJ}}$$

$$Q = mC_p(T_2 - T_1). = mC_p \frac{(P_2V_2 - P_1V_1)}{mR}$$

But $P_1 = P_2$.

$$Q = \frac{C_p}{R} P_1(V_2 - V_1) = \frac{C_p}{R} P_1(2V_1 - V_1)$$

$$Q = \frac{C_p \cdot P_1V_1}{R} = \frac{1.005 \times 128.289}{0.287}$$

$$\boxed{Q = 449.235 kJ}$$

$$\Delta S = mc_p \ln\frac{T_2}{T_1} - mR\ln\frac{P_2}{P_1}$$

$$= mc_p \ln\frac{P_2V_2/mR}{P_1V_1/mR} \quad \left[\text{directly can use } mc_p \ln\frac{V_2}{V_1}\right]$$

$$= mc_p \ln\frac{V_2}{V_1} = 1.5 \times 1.005 \ln 2.$$

$$\boxed{\Delta S = 1.044 kJ/k}$$

(ii) constant volume process (2-3)

$P_3 = 2P_2 = 2P_1 \quad V_2 = V_3.$

$w.D = \int P dV = 0.$

$$Q = mC_v(T_3 - T_2)$$

$$= mC_v\left[\frac{P_3V_3 - P_2V_2}{mR}\right]$$

$$= \frac{C_v}{R}(2P_1 \times V_2 - P_1V_2)$$

$$= \frac{C_v}{R}P_1V_2 = \frac{C_v}{R}P_1 \times 2V_1.$$

$$= \frac{C_v}{R} \times 2 \times 128.289$$

$$= \frac{0.72}{0.287} \times 2 \times 128.289$$

$$\boxed{Q = 643.68 kJ}$$

$$\Delta S = mc_v \ln\frac{T_3}{T_2} + mR\ln\frac{V_3}{V_2}$$

$$\Delta S = mc_v \ln\frac{P_3V_3/mR}{P_2V_2/mR} = mc_v \ln\frac{2P_1V_2}{P_1V_2}$$

$$\Delta S = mc_v \ln 2 \quad \left[mc_v \ln\frac{P_3}{P_2}\right]$$

$$\Delta S = 1.5\times 0.72 \ln 2$$

$$\boxed{\Delta S = 0.748\,kJ/K}$$

$$Q_{net} = 449.285 + 643.68$$
$$= \underline{\underline{1736.595\,kJ}}$$

$$\text{Change in entropy} = \frac{1.044 + 0.748}{}$$

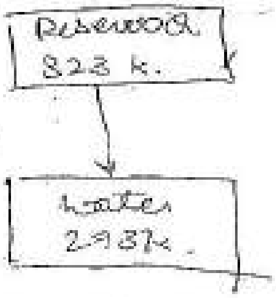
$$\boxed{\Delta S = 1.792\,kJ/K}$$

June 2013

67. one kg of water at 273K is heated to 373K by first bringing it in contact with reservoir at 323K and then reservoir at 373K what is the change in entropy of universe

$$(\Delta S)_w = \int\frac{\delta Q}{T} = mc_p\int_{T_1}^{T_2}\frac{dT}{T} = mc_p\ln\frac{T_2}{T_1}$$

$$(\Delta S)_w = 1\times 4.187\ln\frac{323}{273} = 0.7041\,kJ/K$$



Reservoir temp remains always const

$$Q = mc_p(T_2 - T_1)$$
$$= 1\times 4.187(323-273)$$

$$\underline{Q = 209.35\,kJ}$$

Entropy change of the reservoir.

$$(\Delta S)_{res} = \frac{-Q}{T} = \frac{-209.35}{323} = -0.648 kJ$$

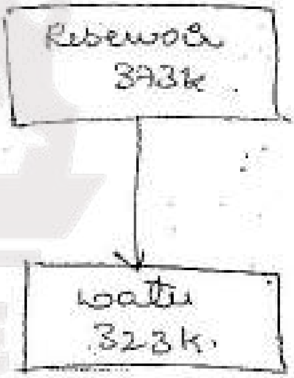
Entropy change of universe = 0.7041 − 0.648

= $\underline{\underline{0.0561 kJ/k}}$

(ii). $\Delta S_w = mcp \frac{T_2}{T_1}$

$= 1 \times 4.187 \ln \frac{373}{323} = 0.6086 kJ/k$.

$(\Delta S)_r = \frac{-Q}{T}$



$Q = mcp(T_2 - T_1)$.

$= 1 \times 4.187 (373 - 323)$.

$Q = \underline{\underline{209.35 kJ}}$.

$(\Delta S)_r = -\frac{209.35}{373} = -0.561 kJ/k$

$(\Delta S)_{univ} = 0.6026 - 0.561$.

$(\Delta S)_{univ} = 0.0416 kJ/k$.

Total change in entropy of universe

$= 0.0561 + 0.0416$.

$(\Delta S)_{univ} = \underline{0.098 kJ/k}$

7). In a shell and tube heat exchanger 45kg of water per minute is heated from 30°C to 85°C by hot gases which enter the heat exchanger at 225°C. If the flow rate of gases is 90kg/min. Find the net change of entropy of the universe.

Given:

$T_{hi} = 225 + 273 = 498K$ $\quad m_h = 90 kg/min$

$T_{ci} = 30 + 273 = 303K$ $\quad m_c = 45 kg/min$

$T_{co} = 85 + 273 = 358K$

Heat lost by hot gases = Heat gained by water

$m_h C_{ph}(T_{hi} - T_{ho}) = m_c C_{pc}(T_{co} - T_{ci})$

$90 \times 1 (225 - T_{ho}) = 45 \times 4.18 (358 - 303)$

$\boxed{T_{ho} = 383K}$

$\Delta S_w = m_c C_{pc} \ln \frac{T_{co}}{T_{ci}}$

$\Delta S_w = 45 \times 4.18 \ln \frac{358}{303}$

$\boxed{\Delta S_w = 31.38 kJ/k}$

$\Delta S_g = m_h C_{ph} \ln \frac{T_{ho}}{T_{hi}}$

$\Delta S_g = 90 \times 1 \ln \frac{383}{498}$

$\boxed{\Delta S_g = -23.63 kJ/k}$

net change in entropy $= \Delta S_w + \Delta S_g$

$= 31.38 - 23.63$

$\boxed{\Delta S_{net} = 7.75 kJ/k}$

8. An adiabatic vessel contains 25kg of oil at a temp of 27°C. A spherical ball made of steel of 10kg at 727°C is immersed in oil. Determine change in entropy for the universe. Take specific heat of oil = 2.5kJ/kgk, specific heat of steel ball = 0.5kJ/kg.

Soln: Given $T_1 = 27 + 273 = 300K$.

$m_{oil} = 85kg$; $m_{steel} = 10kg$

$T_2 = 727 = 1000K$.

$C_{poil} = 3.5 kJ/kgK$, $C_{psteel} = 0.5 kJ/kg$.

Heat lost by steel ball = Heat gained by oil.

$m_s \times C_{ps} (T_2 - T) = m_o C_{po} (T - T_1)$

$10 \times 0.5 (1000 - T) = 85 \times 3.5 (T - 300)$.

$\boxed{T = 311.57K}$

$\Delta S_0 = \int_{T_1}^{T} mC_p \frac{dT}{T}$

$\Delta S_s = \int_{T_2}^{T} mC_p \frac{dT}{T}$

$\Delta S_1 = mC_p \ln \frac{T_1}{T}$

$\Delta S_s = mC_p \ln \frac{T_2}{T}$

$\Delta S_1 = 85 \times 3.5 \ln \frac{300}{311.57}$.

$\Delta S_s = 10 \times 0.5 \ln \frac{1000}{311.57}$.

$\boxed{\Delta S_1 = -11.25 kJ/K}$

$\boxed{\Delta S_s = 5.8 kJ/K}$

change in entropy of universe.

$\Delta S = \Delta S_1 + \Delta S_2$.

$\Delta S = -11.25 + 5.8$

$\boxed{\Delta S = -5.68 kJ/K}$

June Dec 2014

9) A lump of steel of mass 10kg at 627°C is dropped in 100kg of oil at 30°. The specific heats of steel & oil are 0.5kJ/kgK and 3.5 kJ/kgK respectively. calculate the entropy change of steel, the oil & the universe.

Soln: $m_s = 10kg$ $m_o = 100kg$

$T_1 = 627°C = 900K$ $T_2 = 30°C = 303K$

$Cps = 0.5 kJ/kgK$ $Cpoil = 3.5 kJ/kgK$

Heat lost by steel = Heat gained by oil.

$$m_s \times Cp_s (T_1 - T) = m_o \times Cp_o (T - T_2)$$

$$10 \times 0.5 (900 - T) = 100 \times 3.5 (T - 303)$$

$$\boxed{T = 311.4K}$$

$$\Delta S_1 = \int_{T_1}^{T} mcp \frac{dT}{T} \qquad \Delta S_2 = \int_{T_2}^{T} mcp \frac{dT}{T}$$

$$\Delta S_1 = mcp \ln \frac{T}{T_1} \qquad \Delta S_2 = mcp \ln \frac{T}{T_2}$$

$$\Delta S_1 = 10 \times 0.5 \ln \frac{311.4}{900} \qquad \Delta S_2 = 100 \times 3.5 \ln \frac{311.4}{303}$$

$$\boxed{\Delta S_1 = -5.306 kJ/kgK} \qquad \boxed{\Delta S_2 = 0.95 kJ/kgK}$$

$$\Delta S = \Delta S_1 + \Delta S_2$$

$$\Delta S = -5.306 + 0.95$$

$$\boxed{\Delta S = -4.34 kJ/kgK}$$

July 2015

5. In a certain heat exchanger, 50kg of water is heated per minute from 50°C to 110°C by hot gases which enter the heat exchanger at 250°C. If the flow rate of gases is 100kg/min, estimate the net change of entropy. Cp(water) = 4.186 kJ/kgK, Cp(gas) = 1 kJ/kgK.

Given: $m_c = 50kg/min$ $m_h = 100kg/min$

$T_{ci} = 50°C$ $T_{co} = 110°C$

$T_{hi} = 50°C$ $Cp_c = 4.186 kJ/kgK$

$Cp_h = 1 kJ/kg K$

Heat lost by hot gases = Heat gained by water.

$m_hC_{ph}(T_{hi}-T_{ho}) = m_cC_{pc}(T_{co}-T_{ci})$.

$\frac{100}{60}\times 1000\,(250-T_{ho}) = \frac{50}{60}\times 4.186(110-50)$.

$T_{ho} = 124.42°C$.

Entropy change of water

$\Delta S_w = m_cC_{pc}\ln\frac{T_{co}}{T_{ci}}$

$= 50\times 4.186\ln\frac{(110+273)}{(50+273)}$

$\boxed{\Delta S_w = 35.66\,kJ/k}$

$\Delta S_g = m_hC_{ph}\ln\frac{T_{ho}}{T_{hi}}$

$= 1.000\times 1\times\ln\left[\frac{124.42+273}{250+273}\right]$

$\boxed{\Delta S_g = -27.45\,kJ/k.}$

Net change in entropy $= 35.66 - 27.45$

$= \underline{8.21\,kJ/k}$.

July 2015.

1) A piston-cylinder arrangement contains $0.03m^3$ of nitrogen at 1 bar and 290K. The piston moves inwards and the gas is compressed isothermally and reversibly until the pressure becomes 4 bar determine change in entropy and work done. Assume nitrogen to be a perfect gas.

Given: $V_1 = 0.03 m^3$ $P_1 = 1 bar$. $T_1 = 290K$.

$P_1V_1 = P_2V_2$ $P_2 = 4 bar$.

$$V_2 = \frac{1 \times 0.03}{4} = \underline{\underline{0.0075 m^3}}$$

$$\text{work} = P_1V_1 \ln\frac{V_2}{V_1}.$$

$$WD = 1\times10^2 \times 0.03 \ln\frac{0.0075}{0.03}.$$

$$\boxed{WD = -4.1588 \, kJ}$$

change in entropy

$$= \cancel{mC_v \ln\frac{T_2}{T_1}} + mR\ln\frac{V_2}{V_1}$$

$$= mR\ln\frac{V_2}{V_1}$$

$$R_{N_2} = \frac{\bar{R}}{M} = \frac{8.314}{28} = 0.296 kJ/kgK$$

$$\underline{m = \frac{P_1V_1}{RT_1}} = \frac{1\times10^2 \times 0.03}{0.296\times290} = \underline{\underline{0.0348 kg}}$$

$$S_2 - S_1 = -mR\ln\frac{P_2}{P_1}$$

$$S_2 - S_1 = -0.0348 \times 0.296 \ln\frac{4}{1}$$

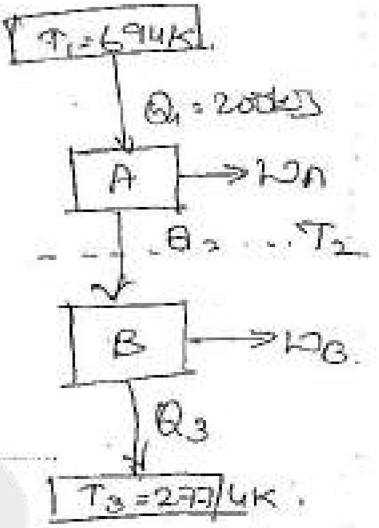
$$\boxed{S_2 - S_1 = 0.01434 kJ/kg}$$

2nd law Dec 2014

25. Two reversible heat engines A & B are arranged in series. A rejecting heat to B through an intermediate reservoir. Engine 'A' receives 200kJ at a tempr of 421°C from a hot source, while engine 'B' is in communication with a cold sink at a tempr of 4.4°C. If the work o/p of A is twice that of B, find (i) the intermediate

temp between A & B. (ii) the efficiency of each engine and (iii) the heat rejected to the cold sink..

Solⁿ: $W_A = 2W_B$.



$$Q_1 - Q_2 = 2(Q_2 - Q_3)$$

$$\frac{Q_1}{Q_2} - 1 = 2\left[1 - \frac{Q_3}{Q_2}\right].$$

$$\frac{T_1}{T_2} - 1 = 2\left[1 - \frac{T_3}{T_2}\right].$$

$$\frac{T_1 - T_2}{T_2} = \frac{2(T_2 - T_3)}{T_2}$$

$$T_1 = T_2 + 2T_2 - 2T_3$$

$$T_2 = \frac{T_1 + 2T_3}{3}$$

$$T_2 = \frac{694 + 2 \times 277.4}{3}$$

$$\boxed{T_2 = 416.26\,K}$$

$$\eta_A = 1 - \frac{T_2}{T_1} = 1 - \frac{416.26}{694} = 40\%$$

$$\eta_B = 1 - \frac{T_3}{T_2} = 1 - \frac{277.4}{416.26} = 33.35\%$$

$$\eta_A = \frac{W_A}{Q_1} \;;\; W_A = \eta_A \times Q_1 = 0.4 \times 200 = \underline{80\,kJ}$$

$$W_B = \frac{W_A}{2} = \frac{80}{2} = 40\,kJ$$

$$Q_1 - Q_2 = W_A.$$

$$Q_2 = Q_1 - W_A = 200 - 80 = \underline{120\,kJ}.$$

$$Q_2 - Q_3 = W_B.$$

$$Q_3 = Q_2 - W_B = 120 - 40 = \underline{80\,kJ}$$

Module 4

Availability, Irreversibility and General Thermodynamic relations.

Introduction, Availability, unavailable energy, relation b/w increase in unavailable energy and increase in entropy, maximum work, maximum useful work for a system and control volume, Irreversibility, second law efficiency (effectiveness). Gibbs & Helmholtz functions, Maxwell relations, clapeyron equation, Joule Thomson coefficient, general relations for change in entropy, enthalpy, internal energy and specific heats

Introduction:

A portion of the heat supplied to any engine must be rejected. No heat engine can have an efficiency equal to 100 percent. The different forms of energy may be divided into two groups.

High grade energy:

Energy that can be completely transformed into shaft work without any loss and hence is fully utilizable. ex: mechanical and electrical work, water, wind and tidal power; kinetic energy of jets etc

Low grade energy:

Energy of which only a certain portion can be converted into mechanical work. ex: heat (or) thermal energy, heat from nuclear fission or fusion

Available and Unavailable energy.

The portion of thermal energy input in cyclic engine which gets converted into mechanical work is referred to as available energy.

The portion of thermal energy which is not utilizable and is rejected to the sink (surroundings) is called unavailable energy.

Lower the temp at which heat is rejected, lesser would be the amount of heat rejected and more would be the work output for a given amount of heat supplied. Lower tempr corresponds to temperature of atmosphere (or a large mass of water (ocean). The available work from a system becomes max when its state is brought in equilibrium to the state of surroundings. This maximum work is known as availability which may be defined as the maximum work available when system comes to equilibrium with its environment by means of work and heat transfer.

Case 1: Heat is withdrawn at constant tempr.

Consider a reversible engine that operates b/w a constant tempr reservoir at temperature T and a sink at tempr To. Corresponding to heat Q supplied by the reservoir, the available work Wmax is given by

$$\eta = \frac{T-T_0}{T} = \frac{W_{max}}{Q}$$

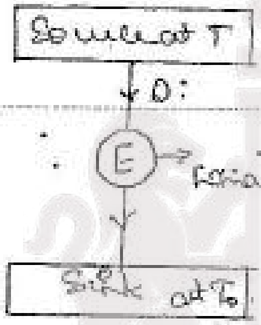
$$\therefore W_{max} = Q\left[\frac{T-T_0}{T}\right]$$

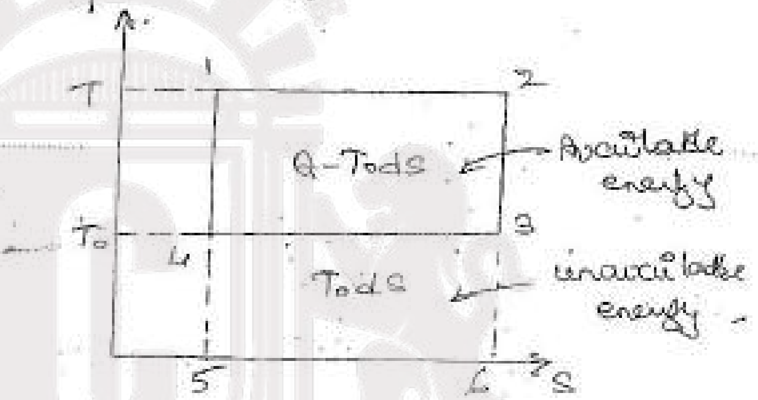
$$= Q\left[1 - \frac{T_0}{T}\right]$$

$$= Q - \frac{Q}{T}T_0$$

$$\boxed{\therefore W_{max} = Q - T_0 dS}$$

where ds represents the change of entropy of the system during the process of heat supply.





Wmax is the availability and on T-S diagram it is given by the area 1-2-3-4. The area 3-4-5-6 represents the unavailable portion of the heat supplied to the engine. Unavailable energy is the energy rejected from the engine, and hence represents the portion of heat supplied that cannot be converted into work. The unavailable energy equals the product of the lowest temperature of heat rejection and the change of entropy of the system during the process of heat supply.

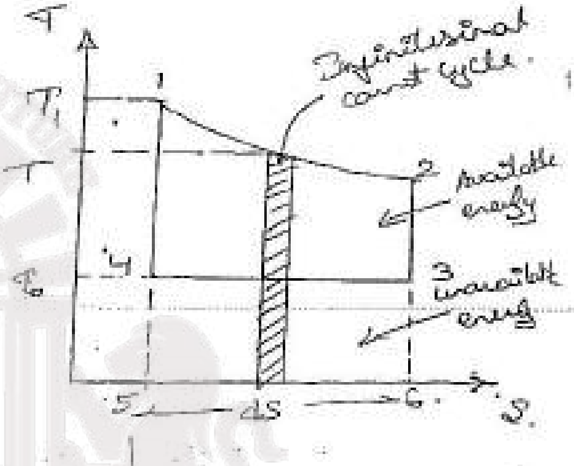
Case 2: Heat is withdrawn at varying temperature

For an finite reservoir, the temperature changes as heat is withdrawn from it, and as such supply of heat to the engine is at varying tempr. The analysis is then made by dividing the process into a series of infinitesimal Carnot cycles each supplying δQ of heat at the tempr T and rejecting heat at the constant tempr T_0. Maximum amount of work then equals

$$W_{max} = \int \left[1 - \frac{T_0}{T} \right] \delta Q$$

$$= \int \delta Q - \int T_0 \frac{\delta Q}{T}$$

$$= \int \delta Q - T_0 \int \frac{\delta Q}{T}$$



$$\therefore \boxed{W_{max} = Q - T_0 \, dS}$$

∴ The expressions for both the available and unavailable are identical in both the cases.

Loss of available energy due to heat transfer through a finite temperature difference

Consider a ~~q~~ certain quantity of heat Q transferred from a system at constant tempr T_1 to another system at const tempr T_2 ($T_1 > T_2$). Before heat is transferred the energy Q is available at T_1 and the ambient tempr is T_0

∴ initial available energy

$$(AE)_1 = Q \left[1 - \frac{T_0}{T_1} \right]$$

After heat transfer, the energy Q is available at T_2 and again the ambient tempr is T_0.

∴ Final available energy.

$$(AE)_2 = Q\left[1-\frac{T_0}{T_2}\right]$$

Change in available energy

$$= (AE)_1 - (AE)_2$$

$$= Q\left[1-\frac{T_0}{T_1}\right] - Q\left[1-\frac{T_0}{T_2}\right]$$

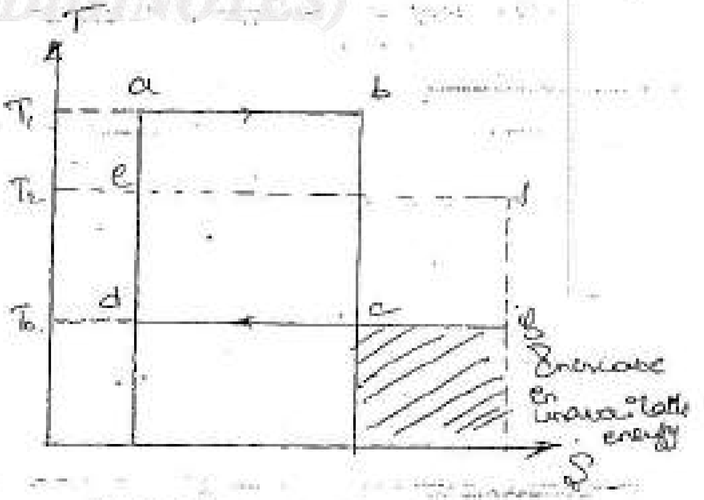
$$= T_0\left[\frac{-Q}{T_1}+\frac{Q}{T_2}\right]$$

$$= T_0\left[dS_1+dS_2\right]$$

$$= T_0(dS)_{net}$$

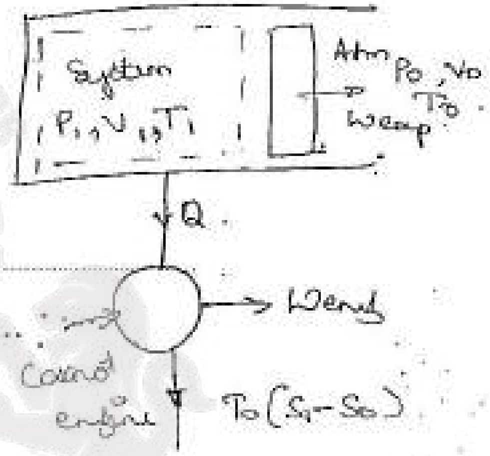
Where $dS_1 = -Q/T_1$, $dS_2 = Q/T_2$ and $(dS)_{net}$ is the net change in the entropy of the comb^n of the two interacting systems. This total entropy change is called entropy of universe or entropy production. Since the heat transfer has been through a finite temp^r difference, the process is irreversible i.e. $(dS)_{net} > 0$ and hence there is loss or decrease of available energy.

Whenever heat is transferred through a finite temp^r diff^n there is always a loss of available energy.



Availability of a non-flow or closed system.

Consider a piston-cylinder arrangement in which the fluid at P_1, V_1, T_1 expands reversibly to the state with parameters P_0, V_0, T_0. The following energy interactions take place.



The fluid expands and expansion work W_{exp} is obtained. From the principle of energy conservation

$$\delta Q = \delta W + dU$$

$$-Q = W_{exp} + (U_0 - U_1)$$

The heat interaction is negative as it leaves the system

$$\therefore W_{exp} = (U_1 - U_0) - Q$$

The heat rejected Q by the piston-cylinder assembly may be made to run a reversible heat engine. The output from the reversible engine equals

$$W_{eng} = Q\left[1 - \frac{T_0}{T_1}\right]$$

$$W_{eng} = Q - T_0(S_1 - S_0) \qquad \left[W_{eng} = \frac{Q_1 - T_0}{T_1}\right]$$

The sum of total of expansion work W_{exp} and the engine work W_{eng} gives maximum work obtainable from the arrangement.

$$W_{max} = \left[(U_1 - U_0) - Q\right] + \left[Q - T_0(S_1 - S_0)\right]$$

$$W_{max} = (U_1 - U_0) - T_0(S_1 - S_0)$$

The piston moving outwards has to spend a work in pushing the atmosphere against its own pressure. This work, which may be called as the surrounding work, is dissipated and is not useful.

$$W_{sur} = p_0(V_0 - V_1).$$

The energy available for work transfer less the work absorbed in moving the environment is called the useful work or net work.

i.e. maximum useful available work or net work.

$$W_{net} = W_{max} - W_{sur}$$

$$= (U_1 - U_0) - T_0(S_1 - S_0) - P_0(V_0 - V_1).$$

$$W_{net} = (U_1 + P_0V_0 - T_0S_0) - (U_0 + P_0V_0 - T_0S_0).$$

$$W_{net} = A_1 - A_0.$$

where $A = (U + P_0V - T_0S)$ is known as non-flow availability function. The system consists of three extensive properties (U, V & S) and two intensive properties of the surroundings (P_0 and T_0)

When the system undergoes a change from state 1 to state 2, without reaching the dead state, then

$$(W_u)_{max} = W_{net} = (A_1 - A_0) - (A_2 - A_0)$$

$$\underline{W_{net} = A_1 - A_2}$$

Availability is a function of the properties of the surroundings as well as the end states of the closed system.

Availability of a Steady flow System

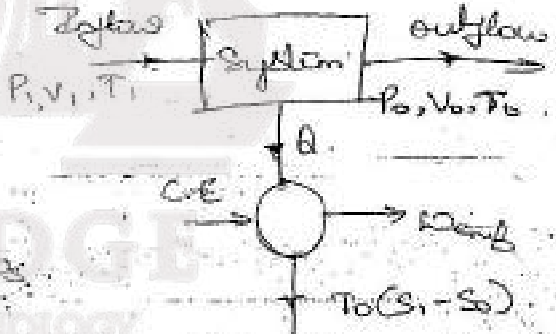
Consider a steady flow system and let it be assumed that the flowing fluid has the following properties and characteristics

Internal energy u, Specific volume v, Specific enthalpy h, pressure p, velocity V and location z.

The properties of the fluid would change when flowing through the system. Let subscript 1 indicates the properties of the system at inlet and subscript 0 be used to fluid parameters at outlet.

Let Q units of heat be rejected by the system and let the system deliver Ws units of work.

The steady flow energy equation may then be written as



$$u_1 + P_1V_1 + \frac{V_1^2}{2} + gZ_1 - Q$$

$$= u_0 + P_0V_0 + \frac{V_0^2}{2} + gZ_0 + W_s$$

neglecting potential & kinetic energy changes,

$$U_1 + P_1V_1 - Q = U_0 + P_0V_0 + W_s$$

$$H_1 - Q = H_0 + W_s$$

$\therefore$ Shaft work $W_s = (H_1 - H_0) - Q$

The heat Q rejected by the system may be made to run a reversible heat engine. The output from this engine equals

$$W_{eng} = Q\left[1 - \frac{T_0}{T}\right] = Q - T_0(S_1 - S_0).$$

∴ maximum available useful work (or net work,

$$W_{net} = W_s + W_{env}$$

$$= (H_1 - H_0) - \cancel{Q} + \cancel{Q} - T_0(S_1 - S_0)$$

$$= (H_1 - T_0 S_1) - (H_0 - T_0 S_0)$$

$$= B_1 - B_0$$

where $[B = H - T_0 S]$ is known as the steady flow availability function. It is a composite property of system and surroundings involving two extensive properties H and S of the system and one intensive property T_0 of the surroundings.

Effectiveness

It represents the fraction of maximum useful work $(W_u)_{max}$ actually utilized and is expressed as the ratio of useful work W_u to the maximum useful work $(W_u)_{max}$.

During cooling (or expansion process, the work produced is at the expense of loss of availability of the system, and the work produced increases the availability of the environment which is given by

$$\varepsilon = \frac{W_u}{(W_u)_{max}}$$

$$\varepsilon = \frac{\text{Gain of availability of the environment}}{\text{Loss of availability of the system}}$$

∴ maximum available useful work (or net work,

$$W_{net} = W_s + W_{env}$$

$$= (H_1 - H_0) - \cancel{Q} + \cancel{Q} - T_0(S_1 - S_0)$$

$$= (H_1 - T_0S_1) - (H_0 - T_0S_0)$$

$$= B_1 - B_0$$

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$$\varepsilon = \frac{W_u}{(W_u)_{max}}$$

$$\varepsilon = \frac{\text{Gain of availability of the environment}}{\text{loss of availability of the system}}$$

The complete work performed by a system W_{exp} is not available for delivery, a certain portion of it has to be spent in pushing out the atmosphere. The work done on the atmosphere equals $P_0 dV$ where P_0 is the atmospheric pressure and dV is the change in volume.

$$W_u = W_{exp} - W_{sur} = W_{exp} - P_0 dV .$$

IIly maximum useful work is

$$(W_u)_{max} = W_{max} - P_0 dV .$$

In a steady flow system, there is no change in the volume of the system, i.e no work is done on the atmosphere.

$\therefore (W_u)_{max} = W_{max}$ in a steady flow.

for a reversible process,

$$(W_u)_{max} = W_u .$$

$\therefore$ the effectiveness is unity.

for a heating (or) compression process.

$$\epsilon = \frac{\text{loss of availability of system}}{\text{gain of availability of the environment}}.$$

Irreversibility.

It denotes the loss of work due to the fact that all real processes are irreversible and is defined as the difference between the maximum work output from the system and the expansion work.

$$I = \left[(U_1 - T_0 S_1) - (U_2 - T_0 S_2)\right] - \left[-Q - (U_2 - U_1)\right]$$

$$\boxed{I = T_0(S_2 - S_1) + Q}$$

Change in entropy of environment due to addition of heat Q at constant atmospheric temperature T_0.

$$(dS)_{surr} = \frac{Q}{T_0} \;;\; Q = T_0 (dS)_{surr}$$

$$\therefore I = T_0\left[(dS)_{sys} + (dS)_{surr}\right]$$

$$I = T_0 (dS)_{net}$$

$$I = T_0 (dS)_{univ}$$

lly for steady flow process.

$$I = \left[(h_1 - T_0 S_1) - (h_2 - T_0 S_2)\right] - \left[(h_1 - h_2) - Q\right]$$

$$I = T_0(S_2 - S_1) + Q$$

$$I = T_0 (dS)_{sys} + T_0 (dS)_{surr}$$

$$I = T_0\left[(dS)_{sys} + (dS)_{surr}\right]$$

$$\boxed{I = T_0 (dS)_{net} = T_0 (dS)_{univ}}$$

Dead State

Problems

1) 1 kg of air is compressed polytropically from a pressure of 1 bar and temp^r of 300K to a pressure of 7 bar and temp^r of 373K. Determine the irreversibility and reversibility measure assuming a sink temp^r of 298K.

Soln: $P_1 = 1\,bar$. $T_1 = 300K$.

$P_2 = 7\,bar$. $T_2 = 373K$.

$T_0 = 298K$.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$\frac{373}{300} = \left(\frac{7}{1}\right)^{\frac{n-1}{n}}$$

$$1.24 = (7)^{\frac{n-1}{n}}$$

$$\boxed{n = 1.13}$$

$$W_{act} = \frac{P_1V_1 - P_2V_2}{n-1} = mR\frac{(T_1 - T_2)}{n-1} = \frac{1\times 0.287(300-373)}{1.13-1}$$

$$\boxed{W_{act} = -162\ kJ/kg}$$

Reversible work $W_r = (U_1 - U_2) - T_0(S_1 - S_2)$

$$= C_v(T_1 - T_2) - T_0\left[C_p\ln\frac{T_1}{T_2} - R\ln\frac{P_1}{P_2}\right]$$

$$W_r = 0.718(300-373) - 298\left[1.005\ln\frac{300}{373} - 0.287\ln\frac{1}{7}\right]$$

$$\boxed{W_r = 153.6\ kJ/kg}$$

Irreversibility = Reversible work − Actual work

$= -153.6 - (-162)$

$= \underline{8.4\ kJ/kg}$

Reversibility measure $= \cancel{\frac{W_{R\,max}}{(W_u)_{max}}}\ \frac{W_u}{(W_u)_{max}} = \frac{-153.6}{-162}$

$\boxed{\text{Reversibility measure} = 0.948}$

2) Air at 5 bar and 500°C expands in a turbine to 1 bar and 290°C. During expansion 12 kJ/kg of heat is lost to the surroundings which is at 0.98 bar and 20°C. Neglecting changes in KE and PE, determine per kg of air

(a) the decrease in availability.

(b) the maximum work required and

(c) the irreversibility.

For air, take $C_p = 1.005$ kJ/kg, $h = c_p \times T$ where C_p is constant

Soln: $P_1 = 5$ bar. $T_1 = 500°C$.

$P_2 = 1$ bar.. $T_2 = 290°C$

$P_0 = 0.98$ bar $T_0 = 20°C$.

Change in availability for a flow process is

$$= B_1 - B_2$$

$$= (h_1 - T_0 S_1) - (h_2 - T_0 S_2)$$

$$= (h_1 - h_2) - T_0 (S_1 - S_2)$$

$$= (h_1 - h_2) + T_0 (S_2 - S_1)$$

$$= C_p (T_1 - T_2) + T_0 \left[C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right]$$

$$= 1.005 (773 - 563) + 293 \left[0.287 \ln \frac{1}{5} - 1.005 \ln \frac{563}{773} \right]$$

$$= 253 \text{ kJ/kg}.$$

Maximum work = Change in availability = 253 kJ/kg.

From steady flow energy equation, neglecting KE & PE

$$Q = h_2 - h_1 + W$$

$$W = Q - (h_2 - h_1)$$

$$W = Q - C_p (T_2 - T_1)$$

$$W = 12 - 1.005 (563 - 773)$$

$$\boxed{W = 223 \text{ kJ/kg}}$$

$\therefore$ Irreversibility $I = W_{max} - W$

$= 253 - 223$.

$$\boxed{I = 30 kJ/kg}$$

3) 0.2kg of air initially at 575K tempr receives 300kJ of heat reversibly at constant pressure. Determine the available and unavailable energies of the heat added. Take cp for air = 1.005kJ/kgK and tempr of surroundings 300K.

$Q = mc_p(T_2 - T_1)$

$300 = 0.2 \times 1.005 (T_2 - 575)$

$\therefore T_2 = \frac{300 + 115.57}{0.201} = \underline{2067.5K}$.

Entropy change.

$dS = mc_p \log_e \frac{T_2}{T_1}$

$= 0.2 \times 1.005 \log_e \frac{2067.5}{575}$

$$\boxed{dS = 0.2572 \ kJ/K}$$

Unavailable work.

$= T_0 dS$

$= 300 \times 0.2572$

$$\boxed{U.W = 77.16kJ}$$

Available work.

$= Q - T_0 dS$

$= 300 - 300 \times 0.2572$

$$\boxed{A.W = 222.84kJ}$$

From Entropy chapter (w.r.to P.S)

$\delta Q - \delta W = dU$.

$\delta Q = TdS$ (from dfn of entropy)

$TdS = dU + PdV$.

w.r.to specific properties

$\boxed{Tds = du + pdv} \rightarrow (i)$

By dfn of enthalpy

$h = u + pv$

$dh = du + pdv + vdp$

(or) $du = dh - pdv - vdp$

Substituting in eqn (i), we have

$Tds = dh - \cancel{pdv} - vdp + \cancel{pdv}$

$\boxed{Tds = dh - vdp} \rightarrow (ii)$

Eqns (i) & (ii) are called as Tds relations
It is used for calculating the entropy changes in reversible process

$$\Delta S = \int dS = S_2 - S_1 = \int \frac{du}{T} + \int \left(\frac{P}{T}\right) dv$$

IIIly

$$\Delta S = \int dS = S_2 - S_1 = \int \frac{dh}{T} - \int \left(\frac{v}{T}\right) dp$$

From Entropy chapter (w.r.to P.S)

$\delta Q - \delta W = dU$.

$\delta Q = TdS$ (from dfn of entropy).

$TdS = dU + Pdv$.

w.r.to specific properties

$\boxed{Tds = du + pdv} \rightarrow (i)$

By dfn of enthalpy

$h = u + pv$

$dh = du + pdv + vdp$

(or) $du = dh - pdv - vdp$.

Substituting in eqn (i), we have

$Tds = dh - \cancel{pdv} - vdp + \cancel{pdv}$

$\boxed{Tds = dh - vdp} \rightarrow (ii)$

Eqns (i) & (ii) are called as Tds relations.
It is used for calculating the entropy changes in reversible process

$$\Delta S = \int dS = S_2 - S_1 = \int \frac{du}{T} + \int \left(\frac{P}{T}\right) dv$$

IIIly

$$\Delta S = \int dS = S_2 - S_1 = \int \frac{dh}{T} - \int \left(\frac{v}{T}\right) dp$$

Calculation of Entropy (w.r. to I.G)

$Tds = du + pdv \longrightarrow (i)$

$Tds = dh - vdp \longrightarrow (ii)$

$ds = \frac{du}{T} + \left(\frac{P}{T}\right)dv$ from relⁿ (i)

For an ideal gas $du = C_v dT$ & $\frac{P}{T} = \frac{R}{v}$.

now by substⁿ

$$ds = \frac{C_v dT}{T} + \left(\frac{R}{v}\right)dv$$

$$\Delta S = \int dS = S_2 - S_1 = \int \left[C_v\left(\frac{dT}{T}\right) + R\left(\frac{dv}{v}\right)\right].$$

$$\Delta S = S_2 - S_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right)$$

from relⁿ (ii)

$$dS = \frac{dh}{T} - \left(\frac{v}{T}\right)dp$$

for an ideal gas $dh = C_p dT$ & $\frac{v}{T} = \frac{R}{P}$.

Subst for $dh = C_p dT$ and $\frac{v}{T} = \frac{R}{P}$

$$\therefore dS = \frac{C_p dT}{T} - \left(\frac{R}{P}\right)dp$$

$$\therefore \Delta S = \int dS = S_2 - S_1 = \int \left[c_p \left(\frac{dT}{T} \right) - \left(R \frac{dp}{p} \right) \right].$$

$$\Delta S = c_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right).$$

July 2013.

Q) A mixture of methane with just enough oxygen to permit combustion, is burned. The temp & pr. of the final mixture are 27°C & 101.3 kPa respectively. Calculate

(i) Mass fraction of reactants.

(ii) The volume fraction of products.

(iii) The partial pressure of water vapour in the products of combination &

(iv) volume of products.

$$CH_4 + 2O_2 \longrightarrow 2H_2O + CO_2$$

$$mf_{CH_4} = \frac{m_{CH_4}}{m} = \frac{1 \times 16}{16 + 2 \times 32} = 0.2$$

$$mf_{O_2} = \frac{64}{16 + 64} = 0.8$$

$$X_{CO_2} = \frac{1}{2+1} = \left(\frac{1}{3} \right) \qquad \therefore X_{CO_2} = \frac{P_{CO_2}}{P}$$

$$X_{H_2O} = \left(\frac{2}{3} \right) \qquad \therefore P_{CO_2} = \underline{\underline{33.\%}}.$$

$$X_{H_2O} = \frac{P_{H_2O}}{P}$$

$$\therefore P_{H_2O} = 6$$

9) 1.25m³ of air at 180°c at 8bar is undergoing a constant pressure until the volume is doubled. Determine the change in entropy & enthalpy of air.

Given $V_1 = 1.25\,m^3$. $V_2 = 2V_1$

$V_2 = 2.5\,m^3$

$P_1 = 8\times10^5\,N/m^2$ $P_1 = P_2$.

$T_1 = 180 + 273 = 453K$

WKT $\Delta H = H_2 - H_1 = mC_p(T_2 - T_1)$

$C_p - C_v = 0.287$ ∴ $C_v = 0.7175$

$C_p/C_v = 1.4$ $C_p = 1.0045$

$PV = mRT$

$m = \frac{PV}{RT} = \frac{1.25\times8\times10^5}{0.287\times453} = 7.69\,kg$

$\Delta H = 7.69\times1.0045\,(453)$

change in entropy.

$S_2 - S_1 = C_p\ln T_2/T_1 + C_p\ln(P_2/P_1)^0$

$S_2 - S_1 = 1.0045\ \ln\left(\frac{906}{453}\right)$

$S_2 - S_1 = 1.0045\ln 2$

$S_2 - S_1 = 0.696\ \text{KJ/kg K}$

MODULE 4

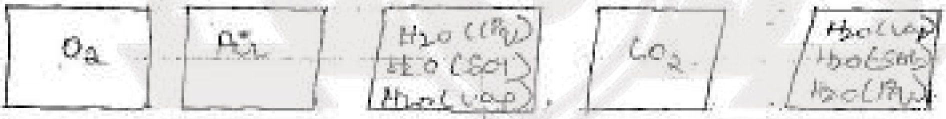
3^{sem} MECH Pure Substances

50

Pure Substance :- It is a substance that has a uniform chemical composition throughout the system e.g. water, N_2, H_2 and CO_2.

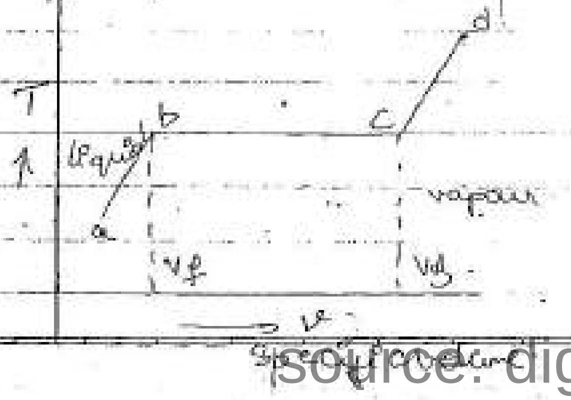
A pure substance is one which has a homogeneous and invariable chemical composition irrespective of the phase (or) phases in which it exists. A mixture of ice, water and steam is a one component system (Pure Subs), since the chemical composition of these three different phases is the same.

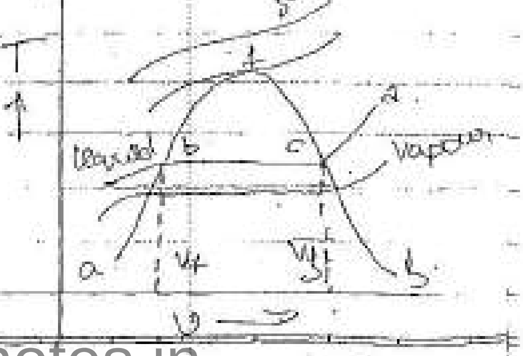
Homogeneous in chemical composition means each part of the system must have the same chemical constituents and the chemical elements must be combined chemically in the same way.



Ex. * To sterilize bottles and cans in dairies
* For large-scale heating such as in community working systems.
* For bleaching in paper & textile industry

Phase diagrams. (T-V) diagram

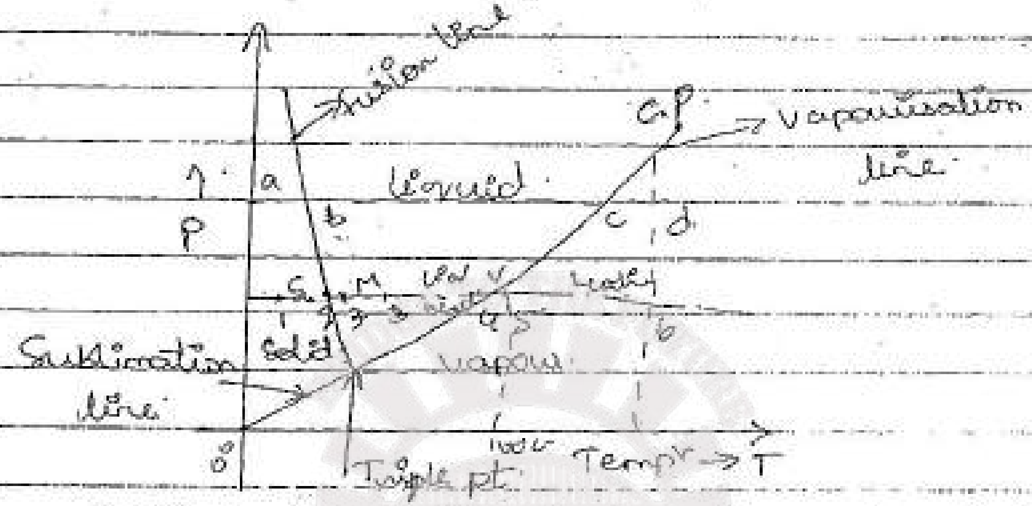




Consider the transfer of heat at constant pressure to a closed system of water. As we see from the fig there is a constant pressure line a-b-c-d which represents the relation b/w the temperature and volume during the course of heat transfer. From a to b, heat transfer produces an increase in temperature and a small increase in volume. When point b is reached, part of water begins to vaporize and a small increase in volume takes place. Point "b" is the liquid state and point "c" is the vapour state. On further heating the system reaches point 'd' of the const. pressure curve.

If the above heating process is repeated at different pressures, curves similar to a-b-c-d are obtained. The points at "b" & "c" we can see there are discontinuities in the slope. If the pressure is increased beyond this critical pressure, the discontinuities disappear. The limiting state at which at which no discontinuities are noticeable is called the critical state & is represented by point 'f'. Line a-b-f is called saturation liquid line & line f-c-g is called saturation vapour line. Both the lines meet at the critical point 'f'. Properties such as temperature, pressure and volume at the critical point are called critical properties. For water critical temperature, pressure, specific volume are 207°C, 224 bar & 0.00061 m^3/kg

Pressure - Temperature Diagram



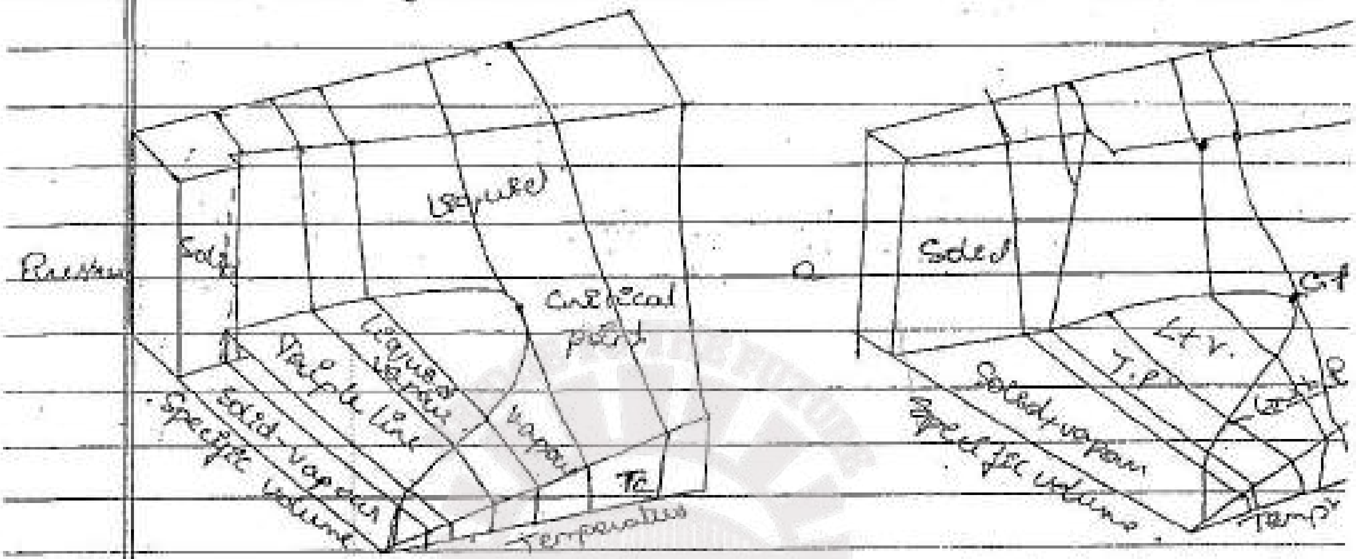
Consider 1kg of ice at a point pressure P and temperature Ta below the fusion temperature Tb. Point b is located on the solid liquid line and determines the state at which the solid is in equilibrium with its liquid.

Heat transfer to the ice at constant pressure results in a temperature increase to Tb. With further transfer of heat, ice begins to liquify at a constant temperature Tb until all the substance is transformed to the liquid state. The transfer of heat to the liquid will increase its temperature to Tc at which evaporation begins, & the temperature remains constant till all the liquid is evaporated.

If the process is repeated at diff^n pressures, the corresponding points b and c can be determined & the solid-liquid and liquid vapour curves can be traced. These two curves meet at a point on the P-T plane called triple point. Heat transfer to a solid will result first in raising its temp^r to the solid vapour line and then changing it directly into vapour. The solid vapour line (sublimation line) represents the state in which the vapour is in equilibrium with its solid.

TP = 273.16 K

P-V-T Surface



The points which are represented on both the T-V & P-T diagrams can be shown on one diagram if the three co-ordinates P, V and T are plotted along rectangular axes. The result is called the P-V-T surface. By projecting the surface on the P-T plane, the whole solid-vapour region projects into the sublimation curve, the whole liquid-vapour region projects into the fusion curve. The critical point is denoted by the letter C_i and the triple point by T. The critical isotherm is marked Tc.

Triple Point

It is that state at which all the three phases of a pure substance coexists in equilibrium. For water, the triple point pressure & temperature are 0.006113 bar and 0.01°C.

Critical Point

The critical point is that state beyond which it is difficult to distinguish b/w liquid and vapour phases. It can also be defined as that state at which liquid can be converted to vapour completely without any heat transfer. (or) It can also be defined as that state at which the liquid - vapour saturation line suddenly terminates. For water, the critical point pressure & temperature are 220.9 bar & 374.14°C.

Steam Generation

The process of steam generation can be divided into three stages

(i) heating of water upto boiling point

(ii) Evaporation of boiling water and its conversion into dry saturated steam

(iii) Transformation of dry saturated steam into superheated steam.

Vapourisation :- It is the process that involves change from the liquid phase to vapour phase water vapours can be obtained by evaporation and steam by boiling.

Evaporation :- It is the process of vapour generation only from the surface of a liquid. Evaporation occurs at any tempr, however its intensity increases with rise in temperature of the liquid.

Saturation :- the degree (or) extent to which something is dissolved (or) absorbed compared with the maximum possible. (or) to a very full extent.

Saturation temperature

Boiling takes place at a definite temperature which depends upon physical properties of the liquid and pressure. This temperature is denoted by t_s is called the saturation temperature & it remains constant until all the liquid has transformed into vapour.

Saturation pressure

The pressure at a specific temperature at which the liquid boils or is in equilibrium with its own vapour is called saturation pressure.

Saturation point / Boiling point

The steam is called saturated when the molecules escaping from the liquid become equal to the molecules returning to it. The saturated steam has a maximum density at a given temperature and is in thermal equilibrium with the liquid. The temperature of saturated steam (vapour) and the liquid in contact with it is called the saturation point or boiling point. The liquid at its boiling point at a specified pressure is called as saturated liquid.

Total heat of water

The amount of heat absorbed by water in being heated upto boiling temperature is known as the total heat of water.

Sensible heat (h_f):

It is the quantity of heat required to raise the temperature of 1kg of water from 0°c to the boiling point as saturation temperature at a given pressure. It is denoted by hf and is also called total heat of water (or enthalpy). Sensible heat increases as the pressure increases.

Latent heat of vapourisation (hfg).

It is the quantity of heat required to convert 1kg of water at saturation temperature for a given pressure into dry and saturated steam at that temperature and pressure. The value of latent heat is not constant, it decreases as the pressure increases and becomes zero when the critical pressure is reached

Wet Steam ⇒

Water particles which are suspended in ~~liquid~~ Steam. Such Steam are known as wet Steam.

When the Steam does not have any suspended water particle is call it as dry Steam.

Dryness Fraction

The dryness fraction of wet Steam is the ratio of the weight of actual dry Steam to the total weight of the wet Steam containing in it.

Dryness fraction = $x = \frac{Ws}{Ws + W}$.

Ws = weight of dry Steam contained in the Steam
W = weight of Suspended particles of water

Total heat of wet Steam.

The amount of heat required to convert, at Constant pressure, 1kg of water at freezing point, into wet steam will be sum of the total heat of water and the latent heat.

$h = h_f + x h_{fg}$

for dry saturated steam.

$h_g = h_f + h_{fg}$

where h_f = Total heat of water
h_{fg} = Latent heat
h = Total heat of steam

Superheated Steam

The heating of steam after it has become dry is known as superheating. The additional heat supplied to the steam during superheating is known as heat of superheating.

Total heat of 1kg of superheated steam
$= h_f + h_{fg} + C_p(T_s - T_1)$

T_1 = Saturation temp at absolute pressure of P,
T_s = Temperature of superheated steam

Volume of wet steam

If x is the dryness fraction of wet steam, then 1kg of this steam will have x kg of dry steam and $(1-x)$ kg of water particles.

volume of 1kg of wet steam = vol of water particles + vol of dry steam.

$= (1-x)V_f + xV_g$

where V_f = volume of 1kg of water
V_g = volume of 1kg of steam.

Internal energy of Steam

The internal energy of Steam is the actual heat energy above the freezing point of water, stored in the Steam, and it may be found by subtracting the external work of evaporation from the total heat.

$U = H - PV_g$ (dry Saturated Steam)

$U = H_x - PxV_g$ (for wet Steam).

$= H_{sup} - PV_{sup}$ (for Superheated Steam)

where, PV_g = The external work/kg of Steam.

Entropy of water

Let 1kg of water be heated at constant pressure from an absolute temperature of T to an absolute temperature of T_1, then change of entropy is

$$= C_p \ln \frac{T_1}{T}$$

The entropy of Steam is always taken above the freezing point of water

$$\therefore \quad S_f = C_p \ln \frac{T_s}{273}$$

Entropy of evaporation

change of entropy $= \int \frac{dH}{T}$

for constant temp^r.

$$\text{change of entropy} = \frac{H}{T}$$

when water is completely evaporated into Steam,

$$\text{Change of entropy} = \frac{h_{fg}}{T}$$

If the Steam is wet, the evaporation will be only partial, & the heat absorbed will be $= \frac{x h_{fg}}{T}$

Entropy of wet Steam

$S_x = S_f + \frac{x h_{fg}}{T}$ (or) $S_x = S_f + x S_{fg}$

Entropy of Superheated Steam :

$= C_p \ln \frac{T_{sup}}{T_{sat}}$

$= S_f + \frac{h_{fg}}{T} + C_p \ln \frac{T_{sup}}{T_{sat}}$ (above freezing point)

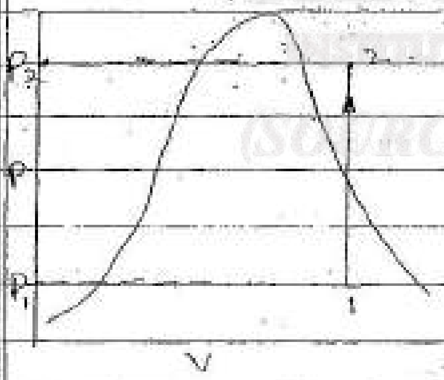
Steam Tables.

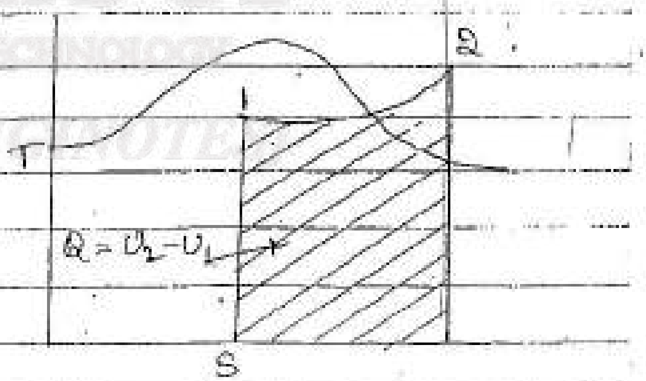
The values of properties of Steam such as pressure, temperature, volume, entropy, enthalpy etc

Absolute	Saturation	specific vol		Enthalpy			entropy	
Pressure	temp	water	Dry	water	latent	Steam	water	Steam
P	T	V_f	V_g	h_f	h_{fg}	h_g	S_f	S_g

Expansion of Vapours

1) constant volume process





Initial volume of steam $= x_1 V_{g1}$

Final volume of steam $= x_2 V_{g2}$

As volume is constant

$x_1 V_{g1} = x_2 V_{g2}$

$\therefore x_2 = \frac{x_1 V_{g1}}{V_{g2}}$

$x_2 < 1$ Steam is wet

$x_2 = 1$ Steam is dry.

$x_2 < 1$ steam is superheat

Thus x_2 can be calculated.

Initial internal energy $U_1 = H_1 - P_1 x_1 V_{g_1}$

Final internal energy $U_2 = H_2 - P_2 x_2 V_{g_2}$

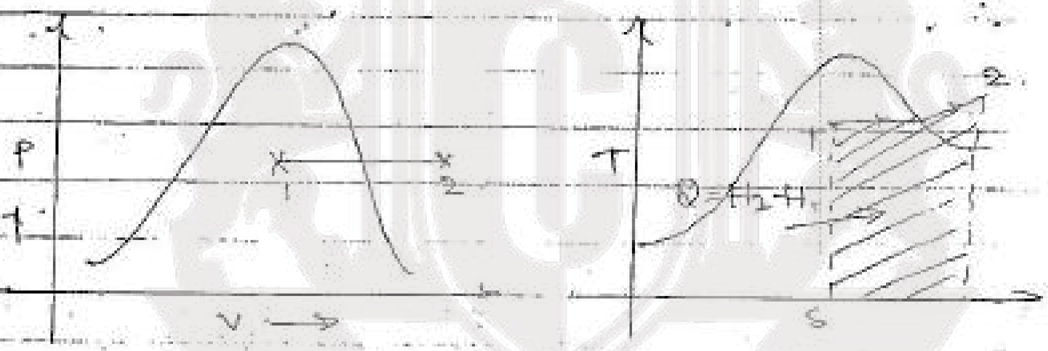
$W_2 = 0$

$\therefore$ Heat transferred Q = workdone + change in internal energy

$= 0 + U_2 - U_1$

$\boxed{Q = U_2 - U_1}$

3) Constant Pressure expansion $(P_1 = P_2)$



Since $P_1 = P_2$, the specific volumes V_{g_1} & V_{g_2} will be same

$\therefore V_{g_1} = V_{g_2} = V_g$

$W.D = Pdv$

$= P(x_2 V_g - x_1 V_g)$

$= P V_g (x_2 - x_1)$

I.E before expn $U_1 = H_1 - P x_1 V_g$

I.E after expn $U_2 = H_2 - P x_2 V_g$

change in IE $(U_2 - U_1) = (H_2 - P x_2 V_g) - (H_1 - P x_1 V_g)$

$= (H_2 - H_1) - P V_g (x_2 - x_1)$

$Q = W + U_2 - U_1$

$= P V_g (x_2 - x_1) + (H_2 - H_1) - P V_g (x_2 - x_1)$

$\boxed{Q = H_2 - H_1}$

3) constant temperature (Isothermal) expansion

Initial volume of steam = $x_1 v_{g1}$

Final volume of steam = $x_2 v_{g2}$

$$P_1 x_1 v_{g1} = P_2 x_2 v_{g2}$$

$$x_2 = \frac{P_1 x_1 v_{g1}}{P_2 v_{g2}}$$

Thus x_2.

If $x_2 = 1$ steam is dry & saturated

$x_2 < 1$ steam is wet

$x_2 > 1$ steam is superheated

Initial internal energy = $U_1 = H_1 - P_1 x_1 v_{g1}$

Final internal energy = $U_2 = H_2 - P_2 x_2 v_{g2}$

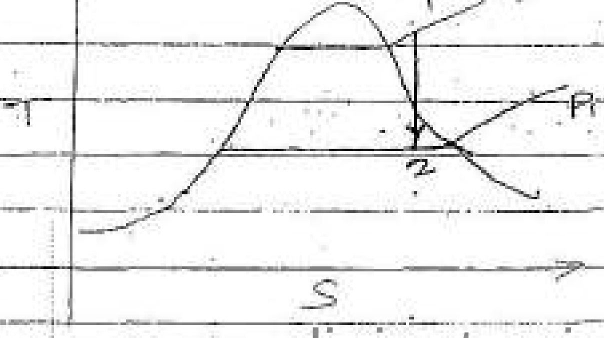
$$U_2 - U_1 = (H_2 - P_2 x_2 v_{g2}) - (H_1 - P_1 x_1 v_{g1})$$

$$U_2 - U_1 = H_2 - H_1$$

workdone $W = P_1 x_1 v_{g1} \ln \frac{x_2 v_{g2}}{x_1 v_{g1}}$

Heat added or rejected. $Q = U_2 - U_1 + W$

4) Reversible adiabatic (or Isentropic) process



Internal energy of steam before expansion: $U_1 = H_1 - p_1 x_1 v_{g_1}$

Internal energy of steam after expansion: $U_2 = H_2 - p_2 x_2 v_{g_2}$

Heat exchange = 0.

$$\therefore\ 0 = W + U_2 - U_1$$

$$\boxed{W = U_1 - U_2}$$

5) Polytropic process

$$pv^n = c.$$

Initial volume of steam $= x_1 v_{g_1}$

Final volume of steam $= x_2 v_{g_2}$

$$p_1 (x_1 v_{g_1})^n = p_2 (x_2 v_{g_2})^n$$

$$x_2 = \frac{x_1 v_{g_1}}{v_{g_2}} \left(\frac{p_1}{p_2}\right)^{1/n}$$

Which gives the final dryness fraction of steam

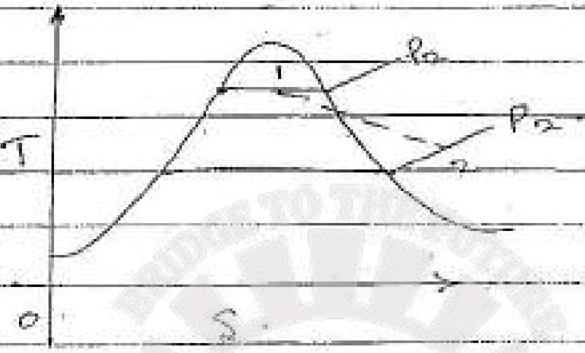
Change in internal energy $= U_2 - U_1$

$$= (H_2 - H_1) - (p_2 x_2 v_{g_2} - p_1 x_1 v_{g_1})$$

$$W = \frac{p_1 x_1 v_{g_1} - p_2 x_2 v_{g_2}}{n-1}$$

$\therefore$ Heat supplied $Q = U_2 - U_1 + W$

5) Throttling process.



Consider 1kg of Steam at a pressure P_1 & dryness fraction x_1 be throttled till its pressure is P_2.

Throttling process occurs when steam is expanded through a small orifice as a nozzle. During this process work is zero & no heat is supplied & therefore total heat remains constant

Total heat before expansion = $h_{f_1} + x_1 h_{fg_1}$

Total heat after throttling = $h_{f_2} + h_{fg_2} + C_p(T_{sup} - T_{sat})$

Total heat before throttling = Total heat after throttling

$$h_{f_1} + x_1 h_{fg_1} = h_{f_2} + h_{fg_2} + C_p(T_{sup} - T_{sat})$$

x_1 can be calculated.

Type of expansion	Equation of condition of steam	work done W	Heat Q
1st Const vol V=C	$x_1 v_{g_1} = x_2 v_{g_2}$	0	$U_2 - U_1$
2nd Const Pre	—	$P v_g (x_2 - x_1)$	$H_2 - H_1$
3rd Const Temp evap / condn	—	$P v_g (x_2 - x_1)$	$H_2 - H_1$
4th Hyperbolic $PV = C$	$P_1 x_1 v_{g_1} = P_2 x_2 v_{g_2}$	$P_1 V_1 \ln V_2/V_1$	$W + U_2 - U_1$
5th Reversible Isentf	$S_1 = S_2$	$U_2 - U_1$	0
6th Polytropic $PV^n = C$	$P_1 (x_1 v_{g_1})^n = P_2 (x_2 v_{g_2})^n$	$\frac{P_1 x_1 v_{g_1} - P_2 x_2 v_{g_2}}{n-1}$	$W + U_2 - U_1$
7th throttling Const. enthalpy (or) total heat	$H_2 - H_1$	0	0

Steam calorimeters.

They are the calorimeters used to determine the quality or dryness fraction of wet steam

Throttling calorimeter.

It is used to determine the dryness fraction when the steam is nearest to the dry condition. It works on the principle that moderate pressure steam has more enthalpy. If high quality of steam at moderate pressure is throttled it tends to become superheated after throttling.

Working

Steam is carried through a sampling tube inserted into a main pipe. It is passed through a constriction in throttle valve that cause drop in pressure without doing external work.

Type of expansion	Equation of condition of vapour	Workdone W	Heat Q
1st Const vol $V=C$	$x_1 v_{g1} = x_2 v_{g2}$	0	$U_2 - U_1$
2nd Const Pre	—	$P v_g (x_2 - x_1)$	$H_2 - H_1$
3rd Const Temp evap/conden	—	$P v_g (x_2 - x_1)$	$H_2 - H_1$
4th Hyperbolic $PV=C$	$P_1 x_1 v_{g1} = P_2 x_2 v_{g2}$	$P_1 V_1 \ln V_2/V_1$	$W + U_2 - U_1$
5th Reversible Isentropic	$S_1 = S_2$	$U_2 - U_1$	0
6th Polytropic $PV^n=C$	$P_1 (x_1 v_{g1})^n = P_2 (x_2 v_{g2})^n$	$\frac{P_1 x_1 v_{g1} - P_2 x_2 v_{g2}}{n-1}$	$W + U_2 - U_1$
7th throttling Const. enthalpy (or) total heat	$H_2 - H_1$	0	0

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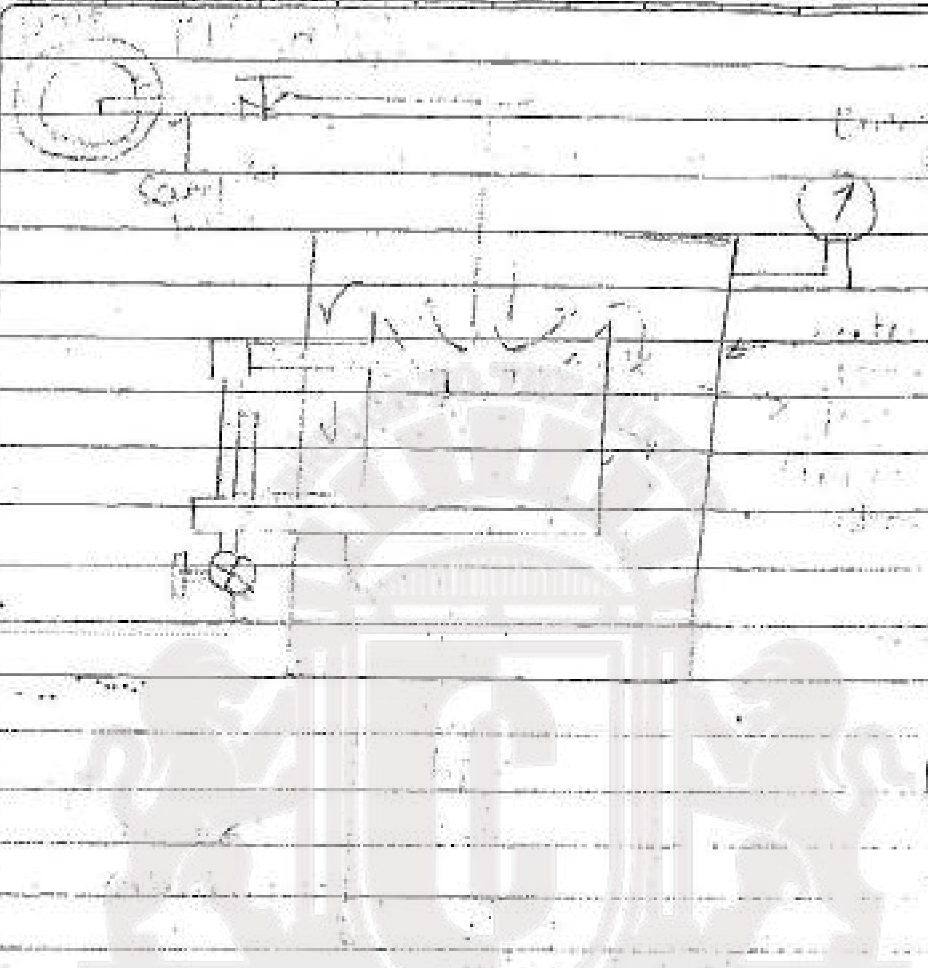
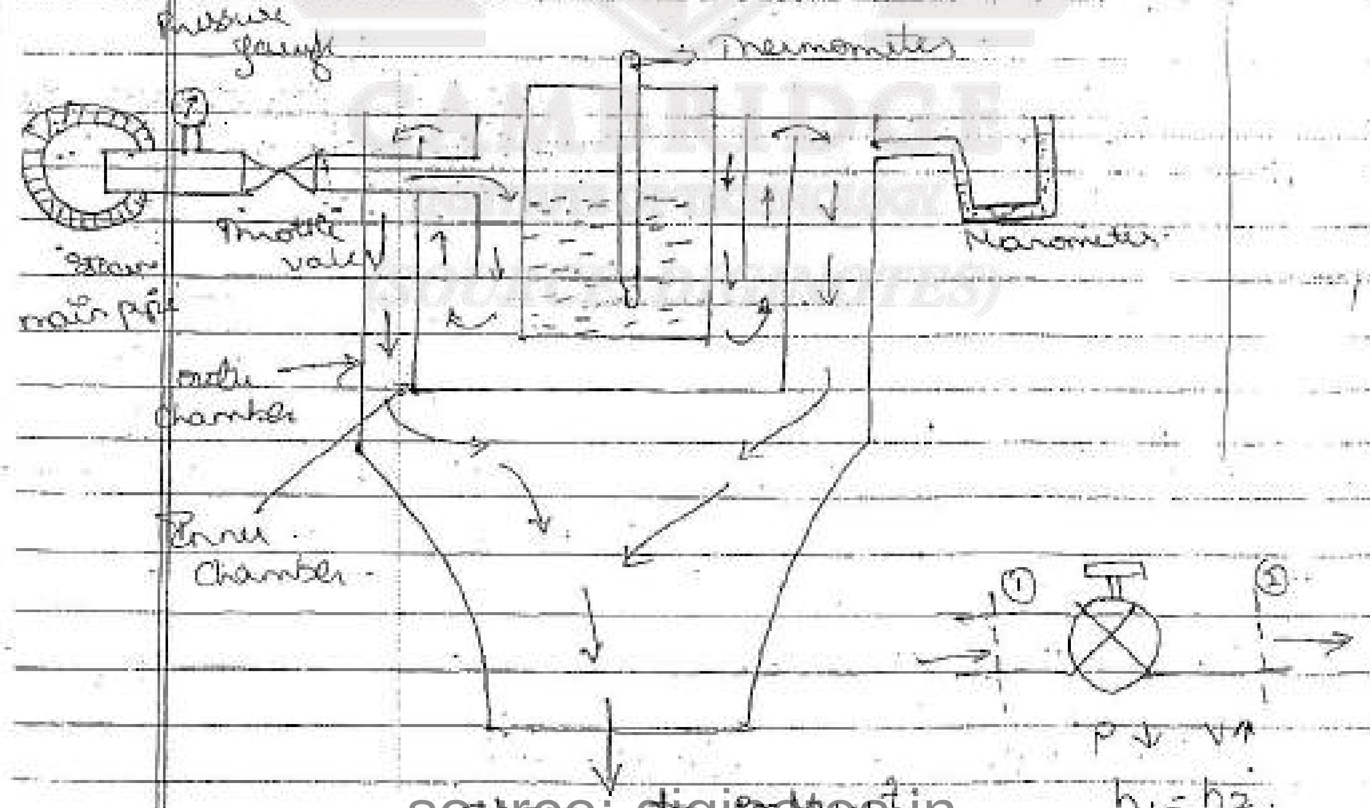



Pressure gauge
Thermometer
Throttle valve
Steam main pipe
Manometer
Outer Chamber
Inner Chamber
①
②

P ↓ V ↑

$h_1 = h_2$.

① A pressure cooker contains 1.5 kg of steam at 5 bar. ~~pressure and 0.4 dry~~. ~~Calculate~~ Find the quality of heat which must be rejected so as to reduce the quality to 60% dry. Determine the pressure & temp of the steam in the new state (Dec 2010) Q.P No 32.

Soln: $m = 1.5\,kg$. Find: $P_2 = ?$

$P_1 = 5\,bar$ $T_2 = ?$

$x_1 = 1$ $Q = ?$

$x_2 = 0.6$ It's a constant <u>volume</u> <u>process</u>

[from 1st Law
$Q = U + W$]

At 5 bar, from steam tables. $t_{sat} = 151.85°C$.

$h_{f_1} = 640.1\,kJ/kg$, $h_{fg_1} = 2107.4\,kJ/kg$, $v_{g_1} = 0.3746\,m^3/kg$

Volume of cooker $\Rightarrow V = m v_{g_1} = 1.5 \times 0.3746$

$$\underline{V_1 = 0.562\,m^3}$$

Initial internal energy $u_1 = h_1 - P_1 v_1$

$$= (h_{f_1} + h_{fg_1}) - P_1 v_{g_1}$$

$$= [640.1 + 2107.4] - 5 \times 10^2 \times 0.3746$$

$$= 2560\,kJ/kg$$

Volume remains same in pressure cooker.

$$V_1 = V_2.$$

$$V_2 = m x_2 v_{g_2}.$$

$$0.562 = 1.5 \times 0.6 \times v_{g_2}.$$

$$\boxed{v_{g_2} = 0.625\,m^3/kg}$$

From Steam tables corresponding to $0.625 m^3/kg$.

$P_2 = 2.9$ bar, $t_{s_2} = 132.4°C$.

$h_{f_2} = 556.5$ kJ/kg, $h_{fg_2} = 2166.6$ kJ/kg.

Final Internal Energy $u_2 = h_2 - P_2 V_2$

$$= (h_{f_2} + x_2 h_{fg_2}) - P_2 x_2 V_{g_2}$$

$$= [556.5 + 0.6 \times 2166.6] - [2.9 \times 10^2 \times 0.6 \times 0.625]$$

$$u_2 = 1747.71 \text{ kJ/kg}$$

Heat transferred at constant volume $= u_2 - u_1$

$$= 1747.71 - 2560$$

$$= -812.29 \text{ kJ/kg}$$

∴ Total heat transferred $= m(u_2 - u_1)$

$$= 1.5(-812.29)$$

$$\underline{= -1218.43 \text{ kJ}}$$

(DEC2011) Q.P NO 5

② A vessel of volume $0.04 m^3$ contains a mixture of saturated water and saturated steam at a tempr of 240°C. The mass of the liquid present is 9 kg. Find the pressure, the mass, the specific volume, the enthalpy, the entropy and the internal energy.

Soln

At 240°C from steam tables.

$P_s = 33.47$ bar, $V_f = 0.0012291$, $V_{fg} = 0.059645$

$h_f = 1037.6$ $h_{fg} = 1764.6$, $h_g = 2802.2$

mass of steam = $\frac{V}{v_g} = \frac{0.04}{0.05964}$ = 0.6706 kg

$V = v_f m_f + v_g m_g$

$x = \frac{m_s}{m_s + m_w} = \frac{0.6706}{9 + 0.6706}$ = ~~0.069341~~ 0.0511

$0.04 = 8 \times 0.001229 + 0.05976 \times m_g$

m_g = 0.5024 kg

$v = v_f + x v_{fg}$

$= 0.001229 + 0.0693 \times (0.059645 - 0.001229)$

$\boxed{v = 0.005277 \text{ m}^3/\text{kg}}$ = 0.004121 m³/kg

sub $\boxed{x = 0.0511}$

$h = h_f + x h_{fg}$

$= 1087.6 + 0.0693 (2802.2 - 1087.6)$

$\boxed{h = 1159.88 \text{ kJ/kg}}$ = 1127.77 kJ/kg

$s = s_f + x s_{fg}$

$= 2.7020 + 0.0693 (6.1406 - 2.7020)$

$\boxed{s = 2.9402 \text{ kJ/kgK}}$ = 2.877 kJ/kg K

$u = h - Pv$

= [illegible]

$= 1159.88 - 33.47 \times 10^2 \times 0.005277$

$\boxed{u = 1158.217 \text{ kJ/kg}}$

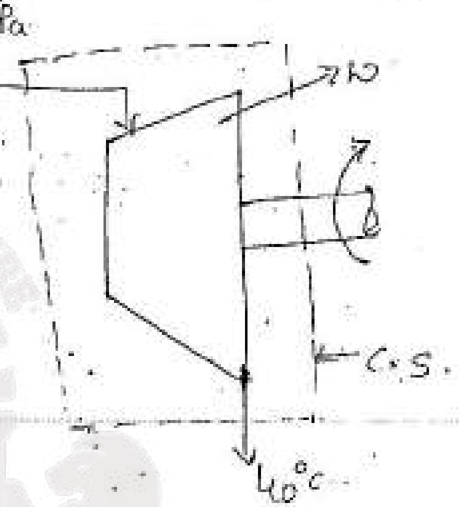
$\boxed{u = 1113.67 \text{ kJ/kg}}$

③ Steam initially at 1.5MPa, 300°C expands reversibly and adiabatically in a steam turbine to 40°C. Determine the ideal work output of the turbine per kg of steam.

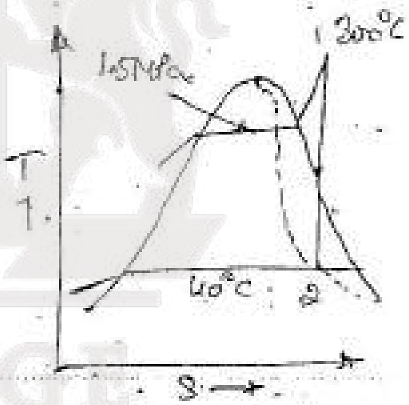
(June 2012) Q.P No 6.

SFEE eqn for turbine

$W = h_1 - h_2$.



∴ work is done by the steam at the expense of a fall in its enthalpy value. The process is reversible and adiabatic, so it is isentropic. [In adiabatic process $S_1 = S_2$].



From steam tables

At 40°C

$P_s = 7.384$ kPa, $S_f = 0.5725$, $S_{fg} = 7.6845$

$h_f = 167.57$, $h_{fg} = 2406.7$ kJ/kg

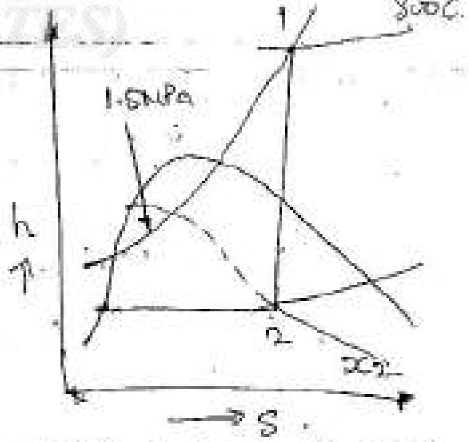
At P = 1.5MPa, t = 300°C,

$S_1 = 6.9189$, $h_1 = 3037.6$ kJ/kg

$S_1 = S_2$

$6.9189 = 0.5725 + x_2(7.6845)$.

$x_2 = 0.826$ or 82.6%



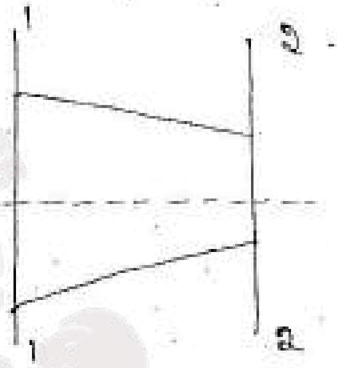
∴ $h_2 = h_f + xh_{fg}$ (40°C).

$h_2 = 167.57 + 0.826 \times 2406.7$

$h_2 = 2152.57$ kJ/kg. [h_1 = at 1.5MPa & 300°C Superheated Steam

$W = h_1 - h_2 = 3037.6 - 2152.57 = 885.03$ kJ/kg

Q Steam at 1MPa & 250°C enters a nozzle with a velocity of 60m/s and leaves the nozzle at 10kPa. Assuming the flow process to be isentropic and the mass flow rate to be 1kg/s. Determine (i) exit velocity (ii) the exit diameter (July 2013), Q.P no 8



Soln: $P_1 = 1MPa$, $T_1 = 250°C$

$P_2 = 10kPa$, $V_1 = 60m/s$.

From steam tables at initial state

$h_1 = 2943.0 \, kJ/kg$, $S_g = 6.9259 = S_2$.

Since its adiabatic process $\boxed{S_1 = S_2}$.

$S_2 = S_f + x_2(S_g - S_f)$

At $P_2 = 0.1 bar$ from S.T.

$x_2 = \frac{S_2 - S_f}{S_g - S_f}$

$S_f = 0.6493$, $S_g = 8.1510$.

$h_f = 191.8$, $h_{fg} = 2392.9$.

$$x_2 = \frac{6.9259 - 0.6493}{8.1510 - 0.6493}$$

$\boxed{x_2 = 0.837}$

$h_2 = h_f + x h_{fg}$

$h_2 = 191.8 + 0.83 \times 2392.9$

$\boxed{h_2 = 2194.6 \, kJ/kg.}$

For a nozzle applying steady flow energy equation

$$(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} \times 10^{-3} = 0.$$

$$V_2 = \sqrt{2(h_2-h_1)10^3 + V_1^2}$$

$$= \sqrt{2(2941.06 - 2194.6) \times 1000 + 60^2}$$

$$= 1225 \text{ m/s}$$

$$v_2 = x_2(v_g - v_f) + v_f$$

from S.T at $P_2 = 0.1$ bar

$$v_2 = 0.0010102 + 0.837(14.67 - 1.0010102)$$

$$v_2 = 12.27 \text{ m}^3/\text{kg}$$

$$m_2 = \frac{A_2 V_2}{v_2}$$

$$A_2 = \frac{m_2 v_2}{V_2}$$

$$A_2 = \frac{1 \times 12.27}{1225}$$

$$\underline{\underline{A_2 = 0.01 \text{ m}^2}}$$

$$A_2 = \frac{\pi}{4} d_2^2$$

$$d_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{4 \times 0.01}{3.142}}$$

$$\underline{\underline{d_2 = 0.1128 \text{ m}}}$$

⑤ Steam at a pressure of 1.6 MPa and 350°C expands according to the $PV^{1.25} = C$ to a pressure of 0.8 MPa. Find (a) the final conditions (b) work done (c) heat transfer and (d) change in entropy. Take mass of the steam as 1 kg.

(Q.B → 6(C))

Solⁿ: At 1.6 MPa & 350°C, $v_1 = 0.17456\ m^3/kg$.

WKT for polytropic process:

$$v_2 = v_1\left[\frac{P_1}{P_2}\right]^{\frac{1}{n}} = 0.17456\left[\frac{1.6}{0.8}\right]^{\frac{1}{1.25}} = 0.30393\ m^3/kg$$

(a) Final conditions and temperature (t_2)

[250] from S.T at 0.8 MPa & $v_2 = 0.30393\ m^3/kg$, it is superheated

at 0.8 MPa, 250°C, $s_1 = 7.0373$, $v_1 = 0.2931$, $h_1 = 2949.3$.

at 0.8 MPa, 300°C, $s_2 = 7.2319$, $v_2 = 0.3241$, $h_2 = 3055.9$.

$$\therefore t_2 = 250 + \left[\frac{\overset{v_2'}{0.30393} - \overset{v_1'}{0.2931}}{\underset{v_2}{0.3241} - 0.2931}\right] \times (300 - 250)$$

$$\boxed{t_2 = 267.46°C}$$

∴ Final condition is superheated

(b) Work done (w) $= \dfrac{P_1v_1 - P_2v_2}{n-1} = \dfrac{10^3(1.6\times0.17456 - 0.8\times0.30393)}{1.25-1}$

$$\boxed{w = 144.608\ kJ/kg}$$

(c) Heat transfer (q)

$$\Delta u = (u_2 - u_1) = (h_2 - P_2v_2) - (h_1 - P_1v_1)$$

$$\Delta u = (\underset{\text{(By interpolation)}}{2986.52} - 0.8\times1000\times0.30393) - (3145 - 1.6\times1000\times0.17456)$$

$$\left[h_2 = 2949.3 + \left[\frac{0.30393 - 0.2931}{0.3241 - 0.2931}\right]\right]$$

$\Delta u = -122.33\ kJ/kg$

$$\therefore q = w + \Delta u = 144.608 - 122.33 = \underline{22.28\ kJ/kg} \quad [3055.9 - 2949.3]$$

(d) $\Delta S = S_2 - S_1$

$\Delta S = (7.1053 - 7.0688)$ (By Interpolation for S_2)
(S_1 at 16 bar).

$$\boxed{\Delta S = 0.03645\ kJ/kgK}$$

⑥ Dry saturated steam contained in a closed vessel undergoes polytropically from 10 bar to 2 bar. The final condition of the steam is wet with dryness fraction 0.9. Calculate (a) the index of expansion and (b) the work done (Q.B → 6(f))

From ST at 10 bar & dry saturated

$$v_1 = v_{g_1} = 0.1944 \, m^3/kg$$

at 2 bar, $v_{g_2} = 0.8859 \, m^3/kg$, $v_{f_2} = 0.0010605 \, m^3/kg$

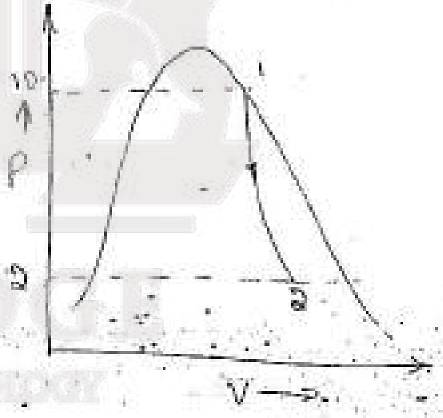
$$v_2 = v_{f_2} + x_2 (v_{g_2} - v_{f_2})$$

$$v_2 = 0.0010605 + 0.9(0.8859 - 0.0010605) = 0.7974 \, m^3/kg$$

(a) Index of expansion (n)

Polytropic process $P_1 v_1^n = P_2 v_2^n$

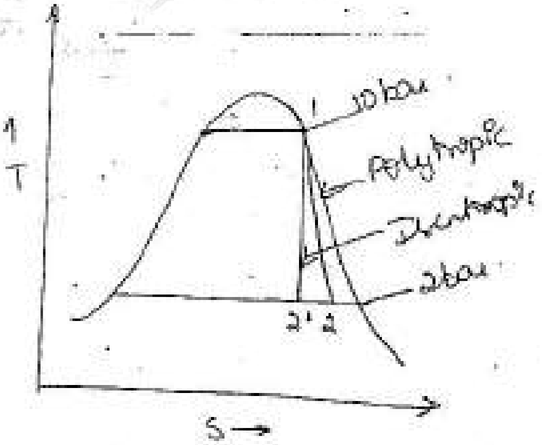
$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{1/n}$$



$$\therefore \ln \frac{P_1}{P_2} = n \log \frac{V_2}{V_1}$$

$$\ln\left(\frac{10}{2}\right) = n \ln\left(\frac{0.7974}{0.1944}\right)$$

$$\boxed{n = 1.1403}$$



(b) The work done (w)

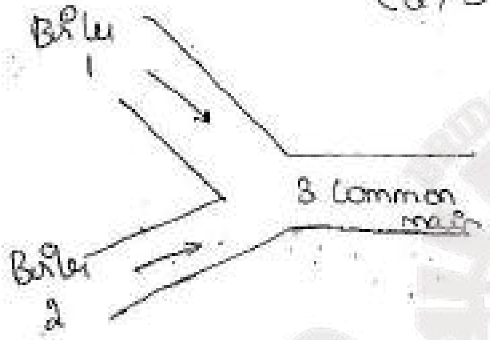
$$w = \frac{(P_1 v_1 - P_2 v_2)}{n-1} =$$

$$= 100 \frac{(10 \times 0.1944 - 2 \times 0.7974)}{1.1403 - 1}$$

$$\boxed{w = 248.895 \, kJ/kg}$$

Steam from boiler 1, at 20bar and 300°C & from boiler 2, at # 20bar enter into a common main. The pressure in the main is 20bar absolute and 250°C. Calculate the quality of the steam supplied from the boiler 2. Cp of superheat is 2.4KJ/kgK

(Q, B ⇒ 6(b))



Boiler 1	Boiler 2	Common main
20 bar.	20 bar	20bar
300°C		250°C

Soln: Assuming each boiler supplies 1kg of steam

Boiler 1 (Superheated steam)

$H_1 = 3022.7 KJ$ (from S.T)

Boiler 2 (Assume steam as wet)

$H_2 = 908.69 + (x_2 \times 1890.0)$ (from S.T)

Common main

From ST at 20bar, saturation temp, ts is 212.42°C, but given is 250°C. Hence steam in common main is superheated.

$H_3 = 2901.6 \times 2 = 5803.2 kJ$

From energy balance

Total enthalpy from boiler 1 + Total enthalpy from boiler 2 = Total enthalpy in common main

$H_1 + H_2 = H_3$

$3022.7 + 908 + x_2 \times 1890.0 = 5803.2$

$\boxed{x_2 = 0.99}$

Q) A rigid vessel is having two compartments, A & B as shown in fig with properties. The partition is removed & the pressure after sometimes is found to be 5 bar. Calculate

(a) The volume of the vessel A & B.

(b) The state of the steam at 5 bar   (Q.B ⇒ 6(h))

(c) The amount of heat transfer

soln:

(a) The volume of the vessel A & B (V_a, V_b)

V_a = volume of A $= m v_a = 1 \times 0.6059 = 0.6059 \, m^3$

V_b = volume of B $= m v_b = 2 \times 0.85 \times 0.2404 = 0.40868 \, m^3$

(b) The state of the steam at 5 bar (x_f)

$V = V_a + V_b = 0.6059 + 0.40868 = 1.0146 \, m^3$

$m = m_a + m_b = 1 + 2 = 3 kg$

v_g at 5 bar $= 0.3749 \, m^3/kg$

$$v_{final} = \frac{V}{m} = \frac{1.0146}{3} = 0.3382 \, m^3/kg$$

Specify volume of the vessel at final condition is 0.3382 is less than v_g at 5 bar $0.3749 m^3/kg$

∴ Steam is wet at final condition

$$v_{final} = [v_f + x_f v_{fg}]_{at\ 5 bar}$$

$$0.3382 = 0.0010925 + x_4\, 0.3738$$

$$\boxed{x_4 = 0.902}$$

(c) The amount of heat transfer from the steam (Q).

x [$H_a = 561.4 + 0.902 \times 2162.2$] x

$u_a =$ ~~[illegible]~~

$u_b =$]

$u_a = h_a + P_1 v_1$

$= 640.1 + 0.902 \times 2107.4 + 3 \times 0.6059$

$u_a = 2543.79\ kJ$

calculation is wrong

x [$u_b = h_b + P_2 v_2$

$= 720.9 + 0.902 \times 2046.5 + 8 \times [0.0011150 + \overset{0.85}{\cancel{0.902}} \times 0.2404]$ x

$u_b = h_b + P_2 v_2$

$= 720.33 + 0.85 \times 1852.3 = 2298.19\ kJ/kg$

$u_b = \overset{m}{2} \times 2298.19 = 4596.3\ kJ$

$u_{final} = h_3 + P_3 v_3 x_3$

$=$ ~~639.81 + 0.902 × 1921.3~~ (from tables at 5bar)

$= 7118.55\ kJ \Rightarrow 3 \times 2372.85$

$(m \times u_4)$

$Q = W + \Delta u$

$= W + (\text{final } U_4)$ $(W=0)$.

$= 7118.5 - (2543.5 + 4596.38)$

$Q = -21.38\ kJ$

9) Steam at a pressure of 8 bar 300°C is throttled to 3 bar & then expanded isentropically to 1 bar. Calculate (a) the change in entropy and (b) the change in enthalpy of these processes $C_{P} = 2.5 kJ/kg$. (Q.B ⇒ 6(i))

Soln: WKT during throttling, enthalpy remains constant.

$$h_1 = h_2 = 3055.9 \, kJ/kg \, , \quad S_1 = 7.2348 \, kJ/kg K \quad (F.T).$$

$$\therefore h_2 = h_{g_2} + C_p [t_{sup} - t_{sat}] \text{ at } 3bar$$

$$3055.9 = 2724.7 + 2.5 [\cancel{300} \, t_{sup} - 133.56]$$

$$\therefore T_2 = t_{sup} = 265.8°C.$$

at 250°C $S = 7.5156 \, kJ/kg K$ at 300 $S = 7.7034 \, kJ/kg K$.

t_{sup} at 3 bar = 265.8°C.

$$S_2 = 7.5156 + \frac{(265.8 - 250)}{(300 - 250)} \times (7.7034 - 7.5156)$$

$$\boxed{S_2 = 7.5744 \, kJ/kg K.}$$

Process 2-3, is isentropic, hence $S_2 = S_3 = 7.5744 \, kJ/kg K$

From steam tables (superheated) at 1 bar

at $t = 100°C$, $h = 2675.9 \, kJ/kg$, $S = 7.3609 \, kJ/kg$.

at $t = 150°C$, $h = 2776.1 \, kJ/kg$, $S = 7.6129 \, kJ/kg$.

at $S_2 = 7.5744 \, kJ/kg K$, h_3 by interpolation

$$h_3 = 2675.9 + \frac{(7.5744 - 7.3609)}{(7.6129 - 7.5744)} \times (7.6129 - 7.3609)$$

$h_3 = 2677.3\ kJ/kg$

(a) The change in entropy $(\Delta S_T)(\Delta S_1)$.

$\Delta S_T = S_2 - S_1$

$= 7.5744 - 7.2319$

$= 0.3425\ kJ/kgK$

$\Delta S_1 = S_2 - S_3 = 0\ kJ/kgK$

(b) The change in enthalpy $(\Delta h_T)(\Delta h_1)$

$\Delta h_T = h_2 - h_1 = 0\ kJ/kg$

$\Delta h_1 = h_3 - h_2 = 2677.3 - 3055.9$

$\Delta h_1 = -378.6\ kJ/kg$

(10) A rigid vessel of 10m³ volume contains steam at 4MPa & 80% quality. Evaluate (a) the enthalpy (b) Internal energy of the steam & (c) entropy of the steam. (Q.B → 6(a))

Solutions. From steam tables at 4MPa :

$h_f = 1087.4\ kJ/kg$, $h_g = 2800.3\ kJ/kg$, $V_f = 0.001252\ m^3/kg$, $V_g = 0.0497\ m^3/kg$, $U_f = 1082.22\ kJ/kg$, $U_g = 2601.5\ kJ/kg$, $S_f = 2.7962\ kJ/kg$, $S_g = 6.0689\ kJ/kgK$.

(a) Enthalpy

$h = h_f + x(h_g - h_f)$

$= 1087.4 + 0.8(2800.3 - 1087.4)$

$\boxed{h = \cancel{2457.}}$ ~~[illegible]~~ $h = 2457.72\ kJ/kg$

$=$ ~~990 X 611.43~~ $=$

(b) Internal energy.

$u = h - pv.$

$v = v_f + x(v_g - v_f).$

$= 0.001252 + 0.8(0.0197 - 0.001252)$

$\boxed{v = 0.04\ m^3/kg.}$

$m = \frac{V}{v} = \frac{10}{0.04} = 250kg.$

$U = m(h) - pv)$

$U = 250[2457.72 - 4 \times 1000 \times 0.04].$

$U = 574.43MJ.$

(c) Entropy

$s = s_f + x(s_g - s_f)$

$= 2.7962 + 0.8(2601.5 - 2.7962)$

$= 5.41436\ kJ/kg\ k.$

$S = 250 \times 5.41436.$

$\boxed{S = 1353.59 kJ/kg}$

⑪ A pressure cooker contains 1.5kg of steam at 5 bar and 0.9 dryness. when the gas was switched off. determine the quality of heat rejected by the pressure cooker when the pressure in the cooker falls to 1bar. (June 2010) QP NO: 1

From S.I. [illegible] $P_2 = 1$ bar

$hf_1 = 639.66$ kJ/kg, $hfg_1 = 1921.5$ kJ/kg. $hf_2 = 417.33$ kJ/kg, $hfg_2 = 2088.7$ kJ/kg

$hg_1 = 2561.2$ kJ/kg, $vg_1 = 0.3749\ m^3$/kg. $vg_2 = 1.694\ m^3$/kg.

$x_1 = 0.9$

$hg_2 = 2506.1$ kJ/kg.

$$x_1 vg_1 = x_2 vg_2.$$

$$x_2 = \frac{0.9 \times 0.3749}{1.694}$$

$$x_2 = 0.199$$

$H_1 = hf_1 + x_1 hfg_1$

$= 639.66 + 0.9 \times 1921.6$

$H_1 = 2369.1$ kJ/kg

$H_2 = hf_2 + x_2 hfg_2.$

$= 417.33 + 0.199 \times 2088.7.$

$H_2 = 832.998$ kJ/kg

$u_1 = H_1 - P_1 x_1 vg_1$

$= 2369.1 - 5 \times 100 \times 0.9 \times 1921.6.$

$u_1 = 2200.39$ kJ/kg

$u_2 = H_2 - P_2 x_2 vg_2$

$= 832.9813 - 1 \times 100 \times 1.694 \times 0.199$

$u_2 = 799.27$ kJ/kg

% Heat transferred $= (u_2 - u_1)$

$= 799.27 - 2200.39$

$= \underline{-1401.12\ \text{kJ/kg}}$

Total heat transfer $= m(u_2 - u_1).$

$= 1.5 \times 1401.12.$

$= \underline{2101.68\ \text{kJ/kg}}$

a) Determine the amount of heat, which should be supplied to 2kg of water at 25°C to convert it into steam at 5bar and 0.9 dry. (Dec 2011) QP no ④.

$m = 2kg, \ T_1 = 25°C, \ x = 0.9$

At 5 bar from S.T

$h_f = 640.12 \ kJ/kg, \ h_{fg} = 2107.38 \ kJ/kg$

$h_2 = h_f + x h_{fg}$

$= 640.12 + 0.9 \times 2107.38$

$= 2536.762 \ kJ/kg$

h_1 = enthalpy of water at 25°C

$= 104.77 \ kJ/kg$

∴ Heat required to convert 1kg of water at 25°C to wet steam $= m(H_2 - H_1)$

$= 2(2536.762 - 104.77)$

$= \underline{4863.984 \ kJ}$

Steam at a pressure of 8 bar 300C is throttled to 3bar and then expanded isentropically to 1 bar. Calculate (a) the change in entropy and (b) the change in enthalpy of these two processes. $C_{pv} = 2.5 kJ/kg$. (Repeat)

WKT during throttling process, enthalpy remains constant

$h_1 = h_2 = 3055.9 \ kJ/kg; \ S_1 = 7.2319 \ kJ/kgK$

From S.T at 8 bar, 300C (Superheated)

$h_1 = 3055.9 \ kJ/kg, \ S_1 = 7.2319 \ kJ/kgK$

$h_2 = h_{g2} + C_p(T_{sup} - T_{sat})$ at 3bar

$$3055.9 = 2725.30 + 2.5\,(t_{sup} - 133.56)$$

$\therefore T_2 = t_{sup}$ at 3bar = $\underline{265.8°C}$ (Repeated problem)

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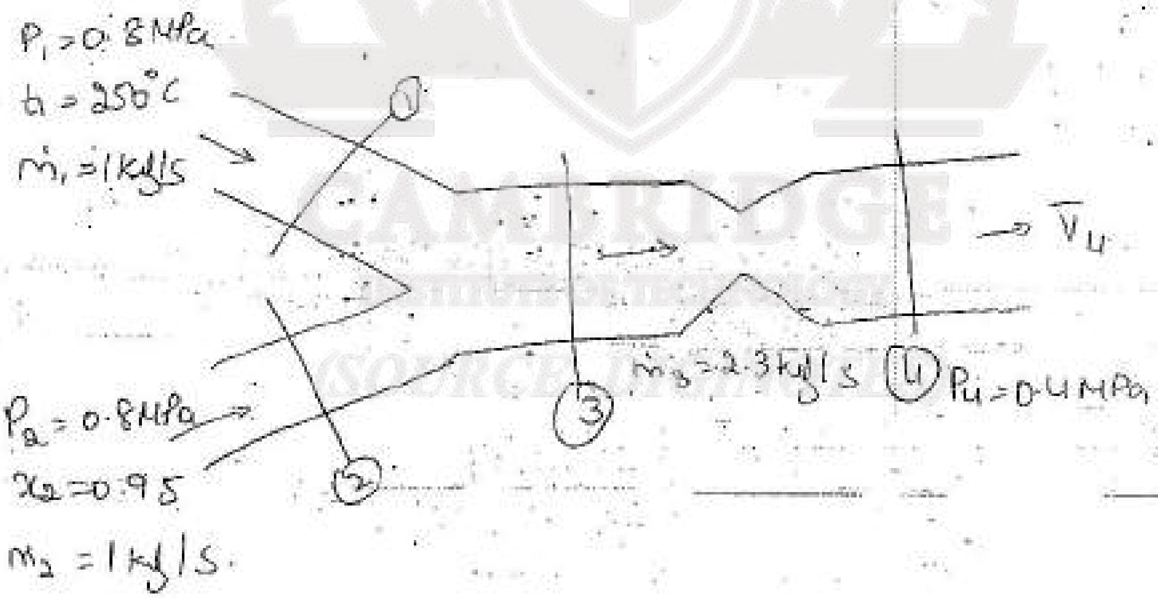
13) Steam at 0.8 MPa, 250°C and flowing at the rate of 1kg/s passes into a pipe carrying wet steam at 0.8MPa 0.95 dry. After adiabatic mixing the flow rate is 2.3kg/s. Determine the condition of steam after mixing. The mixture is now expanded in a frictionless nozzle isentropically to a pressure of 0.4MPa. Determine the velocity of the steam leaving the nozzle. Neglect the velocity of steam in the pipeline. (Q.B) (b)



From fig: $\dot{m}_2 = \dot{m}_3 - \dot{m}_1$

$= 2.3 - 1.0$

$= 1.3$ kg/s

The energy equation for the adiabatic mixing of the two streams gives

$$w_1h_1 + w_2h_2 = w_3h_3 . \quad [m(h_1+h_2) = mh_3].$$

At 0.8MPa, 250°C, $h_1 = 2950.0$ kJ/kg.

At 0.8MPa, 0.95 dry

$$h_2 = h_f + 0.95\, h_{fg}$$

$$= 721.11 + 0.95 \times 2048.0 = 2666.71 \text{ kJ/kg}.$$

∴ $1 \times 2950 + 1.3 \times 2666.71 = 2.3 \times h_3$

∴ $h_3 = 2790$ kJ/kg.

Since $(h_g)_{0.8MPa} = 2769.1$ kJ/kg,

and $h_3 > h_g$, the state must be in the superheated region. ∴ From the S.T, when $p = 0.8$MPa, $t = 200°C$.

$h = 2839.3$ kJ/kg ; $p = 0.8$MPa, $t_{sat} = 170.43°C$

$h_g = 2769.1$ kJ/kg.

By Interpolation

$$t_3 = 179°C.$$

∴ Degree of superheat $= 179 - 170.43 = 8.57°C$

∴ Condition of steam after mixing = 0.8 MPa, 179°C.

The energy eqn for the nozzle gives

$$h_3 = h_4 + \frac{V_4^2}{2}.$$

Since V_3 = - velocity of steam in the pipeline = 0 ..

Steam expands isentropically in the nozzle to 0.4MPa.

By interpolation,

$S_3 = 6.7087$ kJ/kgK = S_4

$6.7087 = 1.7766 + x_4 \times 5.1193$

$x_4 = 0.964$.

∴ $h_4 = 604.74 + 0.964 \times 2133.8 = 2660$ kJ/kg.

$V_4^2 \times 10^{-3} = 2(h_3 - h_4) = 2 \times 130 = 260$

$V_4 = \sqrt{26} \times 100$

$V_4 = 509.9$ m/s

114) A rigid vessel ~~contain~~ of 2m³ volume is filled with superheated steam at 20 bar and 350°C. The vessel is cooled ~~with~~ until the steam is just dry saturated. Calculate the mass of steam in the vessel; the final pressure of steam and amount of energy transferred as heat to the surroundings.

[It's const-volume process]

At 20bar & 350°C

$v_1 = 0.1386\, m^3/kg$ $h_1 = 3138.6$ kJ/kg.

(a) mass of steam (m)

$$m = \frac{2}{v_1} = \frac{2}{0.1386} = 14.43\, kg.$$

(b) The final pressure of steam (P_2).

Rigid vessel, ∴ $V_1 = V_2 = 0.1386\, m^3/kg$.

at $V_2 = 0.1386\, m^3/kg$ & dry saturated $P_2 = 13.98$ MPa
$= 14$ bar.

(c) The amount of energy transferred (Q)

∴ $W = 0$, (Rigid vessel)

$$Q = W + \Delta U = \Delta U = (u_2 - u_1) = (h_2 - P_2 v_2) - (h_1 - P_1 v_1)$$

$$= (3044.6 - 14 \times 1000 \times 0.1386) - (3138.6 - 20 \times 100 \times 0.1386)$$

$$Q = 1101.2 - 366.6$$

$$\boxed{Q = 734.6\, kJ/kg}$$

15) A sample of steam from a boiler drum at 3MPa is put through a throttling calorimeter in which the pressure & tempr are found to be 0.1MPa, 120°C. Find the quality of steam from boiler.

At 20bar from S.T

$h_f = 1008.3$ kJ/kg , $h_{fg} = 1794.0$ kJ/kg.

At 1bar, 120°c.

$h_f = 2675.4$ kJ/kg.

$1008.3 + 1794.0\, x_2 = 2675.4 + 45$ [2.25 × 20]

$$\boxed{\therefore x_2 = 0.954}$$

Following are observations of a throttling & separating calorimeter

Pr in steam main = 15bar abs

Mass of water drained from separator = 0.55kgs

Mass of steam condensed after the throttle = 4.2kgs

Pr & tempr after throttle = 1bar, 120°C

Evaluate x in main

$m_w = 0.55$, $m_s = 4.2$.

$x_1 = \dfrac{4.2 \times x_2}{4.2 + 0.55} = 0.8842\, x_2$.

$h_2 = 2675.4 + 45 = 2720.4$

$h_{f} = (1bar) = 844.6 + x_2 (1945.3) \Rightarrow 15bar$.

$x_2 = \dfrac{2720.4 - 844.6}{1945.3}$

$x_2 = 0.9642 \times 0.8842$

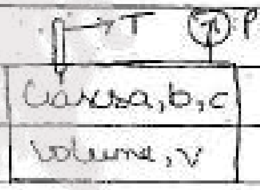
$x_1 = 0.852$

MODULE - 5

Mixture of Gases

The working fluids used in most of the heat engines are not homogeneous gases, but are mixture of gases such as air, combustion products.

The constituent gases are assumed to be ideal, do not interact with one another chemically and the gas mixture obeys the laws of ideal homogeneous gases. Each gas in the mixture behaves independently, occupies the entire volume of the mixture and exerts its own pressure on the walls of the container.



Definitions :-

Consider a mixture of perfect gases a, b, c etc. existing in equilibrium at pressure P and occupying a volume V, at temperature T. Each constituent ~~occupies the same volume that the entire mixture~~ occupies and each constituent is at the same temperature of the mixture

The total mass of the mixture is equal to the sum of the masses of the individual gases

$$m = m_a + m_b + m_c .$$

Mass fraction :- It is the ratio of the mass of the constituent gas to the total mass of the mixture of perfect gases

At 20 bar from S.T.

$h_f = 1008.3$ kJ/kg , $h_{fg} = 1794.0$ kJ/kg.

At 1 bar, 120°C.

$h_f = 2675.4$ kJ/kg.

$1008.3 + 1794.0\, x_2 = 2675.4 + 45$ [2.25 × 20]

$\boxed{\therefore x_2 = 0.954}$

Following are observations in a throttling & separating calorimeter.

Pr in steam main = 15 bar abs.

Mass of water drained from separator = 0.55 kgs

Mass of steam condensed after the throttle = 4.2 kgs

Pr & temp after throttle = 1 bar = 120°C

Evaluate 'x' in main

$m_w = 0.55$, $m_s = 4.2$

$$x_1 = \frac{4.2 \times x_2}{4.2 + 0.55} = 0.8842\, x_2$$

$h_2 = 2675.4 + 45 = 2720.4$

$h_{f2} = (1\ \text{bar}) = 844.6 + x_2(1945.3) \Rightarrow 15\ \text{bar}$

$$x_2 = \frac{2720.4 - 844.6}{1945.3}$$

$x_2 = 0.9642 \times 0.8842$

$x_1 = 0.8524$

$$mf_a = \frac{m_a}{m}, \quad mf_b = \frac{m_b}{m}, \quad mf_c = \frac{m_c}{m}$$

$$mf_a + mf_b + mf_c = 1$$

The sum of the parts of mass fractions of the constituents in a mixture equals to unity.

When an analysis of gas mixture is made on the mass basis, it is called a gravimetric analysis.

Mole fraction

The total number of moles of the mixtures is equal to the sum of the moles of the individual gases.

$$n = n_a + n_b + n_c$$

The mole fraction of any component is the ratio of the number of moles of the constituent gas to the total number of moles of the mixture of perfect gases.

$$x_a = \frac{n_a}{n}, \quad x_b = \frac{n_b}{n}, \quad x_c = \frac{n_c}{n}$$

$$\boxed{x_a + x_b + x_c = 1}$$

Gibb's Daltons law.

Consider a homogeneous non-reacting mixture of ideal gases a, b, c etc at temperature T, pressure P and occupying volume V.

let us assume that each constituent of the mixture exists separately at temperature T and volume V, and pressures Pa, Pb, Pc etc. exerted by individual gases are measured separately.

When the equation of state for an ideal gas is applied to the gas mixture, we have

$$PV = mRT$$

$$PV = nMRT$$

$$P_aV = n_aR_{mol}T$$

$$P_bV = n_bR_{mol}T$$

$$P_cV = n_cR_{mol}T$$

$$(P_a + P_b + P_c + \cdots)V = (n_a + n_b + n_c + \cdots)R_{mol}T$$

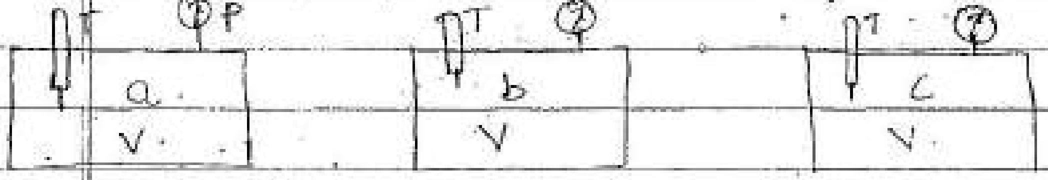
$$= nR_{mol}T$$

$$P = \Sigma P_i \quad \text{where} \quad P_i = \frac{n_iR_{mol}T}{V}$$

represents the pressure the component i would exert. This is called the partial pressure of the ith component of the gas mixture.

Partial Pressure is defined as the pressure which each individual component of a gas mixture would exert if it alone occupied the volume of the mixture at the same temperature.

This is known as Dalton's Law of Partial Pressures.



Amagat's law

It states the sum of the partial volume of the constituent gases is equal to the total volume of the mixture of perfect gases

$$V = V_a + V_b + V_c$$

volume fraction of any constituent gas is defined as the ratio of the volume of the constituent gas to the total volume of the mixture of perfect gases.

$$V_{fa} = \frac{V_a}{V} ; V_{fb} = \frac{V_b}{V} ; V_{fc} = \frac{V_c}{V}$$

$$V_{fa} + V_{fb} + V_{fc} = 1$$

Expression for gas constant R

$PV = mRT$

$P_aV = m_aR_aT ; P_bV = m_bR_bT ; P_cV = m_cR_cT$

But from Dalton's law

$P = P_a + P_b + P_c$

$$\frac{mRT}{V} = \frac{m_aR_aT}{V} + \frac{m_bR_bT}{V} + \frac{m_cR_cT}{V}$$

$$R = \frac{m_a}{m}R_a + \frac{m_b}{m}R_b + \frac{m_c}{m}R_c$$

$$\therefore R = m_{fa}R_a + m_{fb}R_b + m_{fc}R_c$$

Expression for molecular weight M.

The mass of a substance is equal to the product of the number of moles and the molecular weight

$$m = n \times M$$

now $PV = mRT$

$$PV = nMRT \qquad \left\{ \frac{V}{n} = \text{molar Specific volume } \bar{V} \right.$$

$\therefore P\bar{V} = MRT$

Product MR is a constant for all ideal gases and is called the universal gas constant denoted by $\bar{R}$

$$\therefore MR = \bar{R}$$

(or) $P\bar{v} = \bar{R}T$

$$P\frac{V}{n} = \bar{R}T \quad \& \quad PV = n\bar{R}T$$

WKT $\bar{R} = MR \quad \therefore R = \frac{\bar{R}}{M}$

$$\frac{\bar{R}}{M} = mf_a\frac{\bar{R}}{M_a} + mf_b\frac{\bar{R}}{M_b} + mf_c\frac{\bar{R}}{M_c}$$

$$M = \frac{1}{\frac{mf_a}{M_a} + \frac{mf_b}{M_b} + \frac{mf_c}{M_c}}$$

(or)

$m = nM$

$m_a = n_a M_a$; $m_b = n_b M_b$; $m_c = n_c M_c$

$nM = n_a M_a + n_b M_b + n_c M_c$

$M = \frac{n_a}{n} M_a + \frac{n_b}{n} M_b + \frac{n_c}{n} M_c$

$M = X_a M_a + X_b M_b + X_c M_c$

S.T Partial pressure ratio is equal to mole fraction is equal to volume fraction

$PV = mRT$

$PV = nMRT$

$PV = n\bar{R}T$ for the mixture

$P_aV = n_a\bar{R}T$ for the constituent

$\therefore \frac{P_a}{P} = \frac{n_a}{n} = X_a$ = Mole fraction

$PV = n\bar{R}T$ for the mixture

$PV_a = n_a\bar{R}T$ for the constituent.

$\frac{V_a}{V} = \frac{n_a}{n} = X_a$ = Mole fraction

$\therefore \frac{P_a}{P} = X_a = \frac{V_a}{V}$

Similar to June 2012 [illegible] 11

1) A mixture of gases contains 1 kg of CO_2 and 1.5 kg of N_2. The pressure and temperature of the mixture are 3.5 bar and 27°C. Determine for the mixture

1) The mass and mole fraction of each constituent gas.
2) Average molecular weight.
3) The Partial pressures
4) The specific gas constant
5) The volume.
6) The density.
7) The partial volume and volume fraction.

Soln: $m_{CO_2} = 1\,kg$, $m_{N_2} = 1.5\,kg$

$P_1 = 3.5\,bar$, $T_1 = 27°C = 300\,K$

Mass fraction $mf_{CO_2} = \frac{m_{CO_2}}{m} = \frac{1}{(1+1.5)} = 0.4$.

$mf_{N_2} = \frac{m_{N_2}}{m} = \frac{1.5}{(1+1.5)} = 0.6$.

Number of moles $n_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}} = \frac{1}{44} = 0.0227$ kg mole.

$n_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{1.5}{28} = 0.0535$ kg mole.

Mole fraction $X_{CO_2} = \frac{n_{CO_2}}{n} = \frac{0.0227}{(0.0227+0.0535)} = 0.029$

$X_{N_2} = \frac{n_{N_2}}{n} = \frac{0.0535}{(0.0227+0.0535)} = 0.7019$

$R_{mix} = mf_{N_2} R_{N_2} + mf_{CO_2} R_{CO_2}$

$= mf_{N_2} \frac{\bar{R}}{M_{N_2}} + mf_{CO_2} \frac{\bar{R}}{M_{CO_2}}$

$= 0.6 \times \frac{8.314}{28} + 0.4 \times \frac{8.314}{44}$

$R_{mix} = 0.2537$ kJ/kg K

$M_{mix} = \frac{\bar{R}}{R_{mix}} = \frac{8.314}{0.2537} = 32.76$ kg/kg mole

Partial Pressure ratio = mole fraction.

$\frac{P_{N_2}}{P} = X_{N_2}$

$P_{N_2} = P \times X_{N_2} = 3.5 \times 0.7014$

$P_{N_2} = 2.45$ bar.

$P_{CO_2} = P \times X_{CO_2} = 3.5 \times 0.2996$

$P_{CO_2} = 1.0486$ bar

$V_{mix} = \left[\frac{mRT}{P}\right]_{mix} = \frac{2.5 \times 0.2537 \times 300}{3.5 \times 100}$

$V_{mix} = 0.5436 m^3$

Density $= \frac{mass}{Volume} = \frac{2.5}{0.5436} = 4.598$ kg/m^3

Partial volumes $V_{N_2} = \frac{m_{N_2} R_{N_2} T}{P}$

$$= \frac{1.5 \times 8.314}{28} \times \frac{300}{3.5\times10^2}$$

$V_{N_2} = 0.3817 m^3$.

$V_{CO_2} = \frac{m_{CO_2} R_{CO_2} T}{P}$

$$= \frac{1 \times 8.314}{28} \times \frac{300}{3.5\times10^2}$$

$V_{CO_2} = 0.1613 m^3$.

Volume fraction = mole fraction

$V_{f_{N_2}} = X_{N_2} = 0.7014$

$V_{f_{CO_2}} = X_{CO_2} = 0.2976$.

Q) A cubic tank of 1m side contains a mixture of 2kg of N_2 and 3kg of an unknown gas. The mixture pressure and temperature are 280kPa and 350K. Determine

(i) Molecular weight and gas constant of the unknown gas

(ii) The volumetric analysis

Soln: $V = 1m^3$, $m_{N_2} = 2kg$, $m_u = 3kg$

$P = 280kPa$, $T = 350K$.

$$R_{mix} = \left[\frac{PV}{mT}\right]_{mix} = \frac{280\times1}{5\times350} = 0.16 kJ/kgK.$$

$mf_{N_2} = \frac{2}{5} = 0.4 \quad mf_{u} = \frac{3}{5} = 0.6$.

$R_{N_2} = \frac{\bar{R}}{M_{N_2}} = \frac{8.314}{28} = 0.296$ kJ/kg K

$R_{mix} = mf_{N_2}R_{N_2} + mf_u R_u$

$0.16 = (0.4 \times 0.296) + 0.6 \times R_u$

$R_u = 0.0693$ kJ/kg K.

$M_u R_u = \bar{R}$

$M_u = \frac{\bar{R}}{R_u} = \frac{8.314}{0.0693} = 119.97$ kg/kg mole.

The number of moles

$n_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{2}{28} = 0.07142$ kg mole.

$n_u = \frac{m_u}{M_u} = \frac{3}{119.97} = 0.025$ kg mole

Total number of moles $n = 0.07142 + 0.025$

$n = 0.09642$ kg mole.

Mole fraction $X_{N_2} = \frac{n_{N_2}}{n} = \frac{0.07142}{0.09642} = \underline{0.7407}$

$X_u = \frac{n_u}{n} = \frac{0.025}{0.09642} = \underline{0.2592}$

But volume fraction = mole fraction.

$V_{fN_2} = 0.7407$, $V_{fu} = 0.2592$

∴ Volumetric composition will be

$N_2 = 74.07\%$ Unknown gas = 25.92%.

3) A gaseous mixture has the following volumetric analysis $O_2 = 30\%$, $CO_2 = 40\%$, $N_2 = 30\%$, determine

a) the analysis on mass basis

b) molecular weight of the mixture &

c) The partial pressure of each component if the total pressure is 100kPa and temperature is 32°C.

Solⁿ: $V_{fO_2} = 0.3$, $V_{fCO_2} = 0.4$, $V_{fN_2} = 0.3$.

WKT partial pressure ratio = volume fraction

$$\therefore \frac{P_{O_2}}{P} = V_{fO_2}$$

$P_{O_2} = P \times V_{fO_2} = 100 \times 0.3 = 30\,kPa$

$P_{CO_2} = P \times V_{fCO_2} = 100 \times 0.4 = 40\,kPa$

$P_{N_2} = P \times V_{fN_2} = 100 \times 0.3 = 30\,kPa$

Again mole fraction = volume fraction

$X_{O_2} = V_{fO_2}$.

Molecular weight of the mixture =

$X_{O_2}M_{O_2} + X_{CO_2}M_{CO_2} + X_{N_2}M_{N_2}$

$(0.3\times32) + (0.4\times44) + (0.3\times28)$

$M_{mix} = 35.6$ kJ/kg mole

$$X_{O_2} = \frac{n_{O_2}}{n} = \frac{m_{O_2}/M_{O_2}}{m/M_{mix}}$$

$$= \frac{m_{O_2}}{m}\times\frac{M_{mix}}{M_{O_2}}$$

$$= mf_{O_2}\times\frac{M_{mix}}{M_{O_2}}$$

$$\therefore mf_{O_2} = X_{O_2}\frac{M_{O_2}}{M_{mix}} = \frac{0.3\times32}{35.6} = 0.2696$$

$$mf_{CO_2} = X_{CO_2}\frac{M_{CO_2}}{M_{mix}} = \frac{0.4\times44}{35.6} = 0.4943$$

$$mf_{N_2} = X_{N_2}\frac{M_{N_2}}{M_{mix}} = \frac{0.3\times28}{35.6} = 0.2396$$

Mass analysis of the mixture is

$O_2 = 26.96\%$.

$CO_2 = 49.43\%$.

$N_2 = 23.95\%$.

Q.P. Dec 2011

Q. Specific heats of a gas are given by $C_p = a + KT$ & $C_v = b + KT$, where a, b & K are constants & T is in K. 1.5 kg of this gas occupying a volume of $0.06 m^3$ at 5.6 MPa, expands isentropically until the temp^n is 240°C. If $a = 0.946$, $b = 0.662$ & $K = 10^{-4}$. calculate the workdone in the expansion

Soln. $C_p - C_v$

$a + \cancel{KT} - b - \cancel{KT}$

$a - b = R$

WKT $ds = C_v \frac{dT}{T} + R \frac{dv}{v}$

$= (b+KT)\frac{dT}{T} + (a-b)\frac{dv}{v}$

$ds = b\frac{dT}{T} + K dT + (a-b)\frac{dv}{v}$

For an isentropic process:

$b \ln T + KT + (a-b) \ln v = \text{Const}$

$T^b \, v^{a-b} \, e^{KT} = \text{Const}$

not reqd

(b) $R = a - b = 0.946 - 0.662 = 0.284 \text{ kJ/kg K}$

$T_2 = 240 + 273 = 513 K$

$T_1 = \frac{P_1 V_1}{mR} = \frac{5.6 \times 10^3 \times 0.06}{1.5 \times 0.284} = \underline{789 K}$

$\therefore W_2 = -\int_{T_1}^{T_2} m C_v \, dT$

$W_2 = 1.5 \int_{513}^{789} (0.662 + 0.0001T) dT$

$W_{1-2} = 1.5[0.662(789-573) +$

$10^{-4} \times 0.5 \{(789)^2 - (573)^2\}]$

$W_2 = 1.5(182.71 + 19.97)$

$\boxed{W_2 = 304\,kJ}$

July 2016.

5) A mixture of ideal gases consists of 3kg of N_2 and 54kg of CO_2 at a pressure of 200kPa and a temp of 20°C. Find

(i) Mole fraction of each constituent.
(ii) The equivalent molecular wt of mixture
(iii) The equivalent gas constant of the mixture
(iv) The partial pressure of each gas

Sol: $X_{N_2} = \frac{n_{N_2}}{\Sigma n_{N_2}} = \frac{\frac{3}{28}}{\frac{3}{28} + \frac{54}{44}} = \frac{0.107}{0.107+1.23}$

$X_{N_2} = 0.0802$

$X_{CO_2} = \frac{\frac{54}{44}}{\frac{3}{28} + \frac{54}{44}} = 0.9199$

b) Equivalent molecular wt of the mixture

$$M = x_{N_2} \times M_{N_2} + x_{CO_2} \times M_{CO_2}$$

$$M = 0.0802 \times 28 + 0.9149 \times 44$$

$$\boxed{M = 42.72\ kg/kgmol}$$

c) Equivalent gas const of the mixture

$$R = \frac{m_{N_2} R_{N_2} + m_{CO_2} R_{CO_2}}{m}$$

$$R = \frac{3 \times \frac{8.314}{28} + 54 \times \frac{8.314}{44}}{57} = \frac{0.8907 + 10.203}{57}$$

$$\boxed{R = 0.19\ kJ/kg\,K}$$

(d) Partial pressure of each gas.

$$\frac{P_{N_2}}{P} = x_{N_2} ; \quad 0.0802 \times 300 = \underline{\underline{24.06\ kPa}}$$

$$\frac{P_{CO_2}}{P} = x_{CO_2} ; \quad 0.9199 \times 300 = \underline{\underline{275.7\ kPa}}$$

Q) 5kg of argon gas undergoes a change of state at constant internal energy. The initial pressure and tempr are 6 atm & 300K respectively. The final volume occupied by the gas is 3 times that occupied initially. Assume ideal gas behavior, 20

Determine :

(i) the final temp^r of the gas

(ii) the final pr of the gas

(iii) the entropy change of the gas due to change of state.

Given : m = 5gm $P_1 = 6atm$

$T_1 = 300K$. $V_2 = 8V_1$

$N = 40$

$$R = \frac{\bar{R}}{N} = \frac{8.314}{40} = 0.20785 \text{ kJ/kgK}$$

Since internal energy is constant

$$T_2 = T_1 = 300K$$

$P_1V_1 = mRT_1$

$$V_1 = \frac{5\times10^{-3}\times0.20785\times300}{6\times10^{2}}$$

$$\boxed{V_1 = 5.19625\times10^{-3}\ m^3}$$

$V_2 = 3V_1 = 3\times5.1962\times10^{-3}$

$$\boxed{V_2 = 0.015588\ m^3}$$

$$P_2 = \frac{mRT_2}{V_2} = \frac{5\times10^{-3}\times0.20785\times300}{0.015588}$$

$$\boxed{P_2 = 200kPa}$$

$$S_2 - S_1 = \cancel{mc_v \ln\frac{T_2}{T_1}}^{\,0} + mR \ln\frac{V_2}{V_1}$$

$$S_2 - S_1 = 5\times10^{-3}\times0.20785 \ln 3$$

$S_2 - S_1 = 1.14$ [illegible] $\times10^{-3}$ kJ/K

7) A gas at a pressure of 1.4 MN/m² & 360° is expanded adiabatically to a pressure of 100 kN/m². The gas is then heated at constant volume until it attains 360°C when its pressure is found to be 220 kN/m². Finally it is compressed isothermally to the original pressure of 1.4 MN/m². Sketch the process on P-V & T-S diagrams. For 0.23 kg of gas, evaluate the following work transfer, heat transfer, change in internal energy & change in entropy during each process. Assume the following data for the gas. $C_p = 1.005$ kJ/kgK, $C_v = 0.718$ kJ/kgK.

$R = 0.287$ kJ/kgK, $\gamma = 1.4$.

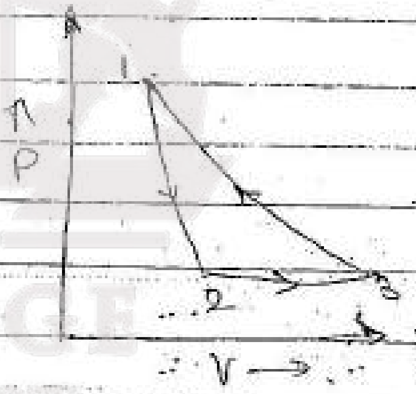
Soln: $P_1 = 1.4\ MN/m^2 = 1400\ kPa$

$T_1 = 360°C$

$P_2 = 100\ kN/m^2 = 100\ kPa$

$T_3 = 360°C$

$P_3 = 220\ kPa$



Process 1-2 adiabatic

2-3 Const volume

3-1 Isothermal

$C_p = 1.005$ $C_v = 0.718$

$R = 0.287$ $\gamma = 1.4$

$m = 0.23\ kg$

Process 1-2 adiabatic

$_1Q_2 = 0$

$$_1W_2 = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 683\left(\frac{100}{1400}\right)^{\frac{1.4-1}{1.4}}$$

$$\boxed{T_2 = 297.82K}$$

$$_1W_2 = \frac{0.23 \times 0.287\,(683-297.82)}{1.4-1}$$

$$\boxed{_1W_2 = 22.1kJ}$$

$$_1Q_2 = U_2 - U_1 + {_1W_2}$$

$$U_2 - U_1 = -22.1kJ$$

$$S_2 - S_1 = 0$$

Process 2-3 Constant volume

$$_1W_2 = 0$$

$$U_2 - U_1 = mC_v(T_3 - T_2)$$

$$= 0.23 \times 0.718\,(633-297.82)$$

$$\boxed{U_2 - U_1 = 55.35kJ}$$

$$_1Q_2 = (U_2 - U_1) = \underline{55.35kJ}$$

$$S_3 - S_2 = mC_v \ln\frac{T_3}{T_2} + mR\ln\frac{V_3}{V_2}$$

$$= 0.23 \times 0.718 \ln\frac{633}{297.82} + 0.$$

$$\boxed{S_3 - S_2 = 0.1265 kJ/K}$$

Process 3-1 Isothermal

change in internal energy = $U_1 - U_3 = 0$.

Work = $_3W_1 = P_3V_3 \ln \frac{V_1}{V_3} = mRT_3 \ln \frac{P_3}{P_1}$

$_3W_1 = 0.23 \times 0.287 \times 633 \ln \frac{220}{1400}$.

$\boxed{_3W_1 = -77.32 kJ}$

∴ Heat transferred | $_3Q_1 = -77.32 + 0$.

$_3Q_1 = -77.32 kJ$.

change in entropy = $S_1 - S_3$

$= mc_p \ln \frac{T_3}{T_1} - mR \ln \frac{P_3}{P_1}$.

$S_1 - S_3 = -0.23 \times 0.287 \ln \frac{220}{1400} + 0$

$\boxed{S_1 - S_3 = 0.122 kJ/k}$

REAL GASES.

A real gas is one which does not obey the ideal gas equation PV=RT at all temperatures and pressures.
No real gas actually follows the ideal gas equation of state at all ranges of pressure and temperature.
The behaviour of real gas approaches closely that of the ideal gas provided the following assumptions are valid,

- Any finite volume of a gas consists of a very large number of molecules.
- The molecules are separated by distances large compared to their own dimensions & are in a state of continuous motion.
- Molecules exert no force on one another except when they collide.
- collisions of molecules with one another & with the walls of the container are perfectly elastic.

The deviation from ideal gas behaviour at a given temperature & pressure can be accurately predicted by the introduction of a correction factor called the compressibility factor.

This factor denoted by Z, is defined as

$$Z = \frac{PV}{RT}$$

For an ideal gas Z=1, for a real gas the value of Z is a measure of the deviation of the real gas from the ideal gas

Vander waals equation

Real gases differ from ideal ones due to presence of the Intermolecular forces and also to the finite molecular volumes. Thus are two corrections into the equation of state, $PV = RT$,

(i) let the equation of state be presented in the form, $V = RT/P$, with increasing pressure, the volume diminishes and if $P \to \infty$, then $V \to 0$. The molecules occupy a finite volume and an allowance also needs to be made for the volume of voids existing between molecules. The free volume available for molecular motion shall be equal to $(V - b)$, where $b = V_{mol} + V_{void}$. The quantity b represents the smallest volume to which the gas can be compressed.

(ii) compared to an ideal gas, the pressure exerted by a real gas will be smaller by a quantity ΔP. This pressure is directly proportional to the square of the density of gas, (or) inversely proportional to the square of its specific volume.

$$\Delta P = a/V^2 = a\rho^2$$

When these two corrections are introduced, we obtain the Vander waal's equation of state.

$$\left[P + \frac{a}{V^2}\right][V - b] = RT.$$

b ⇒ to correct for the volume occupied by the gas molecules

a/V^2 ⇒ is a correction for the molecular forces of attraction.

Determination of Vander waals constants

$$P_c = \frac{RT_c}{V_c - b} - \frac{a}{V_c^2}$$

$$\frac{dP}{dV} = \frac{-RT_c}{(V_c-b)^2} + \frac{2a}{V_c^3} = 0$$

(or) $RT_c = \frac{2a}{V_c^3}(V_c-b)^2 \longrightarrow (i)$

$$\frac{d^2P}{dV^2} = \frac{2RT_c}{(V_c-b)^3} - \frac{6a}{V_c^4} = 0$$

(or) $RT_c = \frac{3a}{V_c^4}(V_c-b)^3 \longrightarrow (ii)$

$$\frac{2a}{V_c^3}(V_c-b)^2 = \frac{3a}{V_c^4}(V_c-b)^3$$

(or) $2 = \frac{3(V_c-b)}{V_c} = \frac{3V_c - 3b}{V_c}$

(or) $2V_c = 3V_c - 3b$

$\boxed{V_c = 3b} \longrightarrow$

Subs $V_c = 3b$ in eqn (i).

$$RT_c = \frac{2a}{27b^3}(3b-b)^2$$

$$= \frac{2a \times 4b^2}{27b^3} = \frac{8a}{27b}$$

$\therefore T_c = \frac{8a}{27bR}$

By substituting the values of V_c & T_c in Vander waal's eqn, we get,

$$P_c = \frac{R(8a/27bR)}{(3b-b)} - \frac{a}{9b^2} = \frac{a}{27b^2}$$

$$\therefore \boxed{P_c = \frac{a}{27b^2}}$$

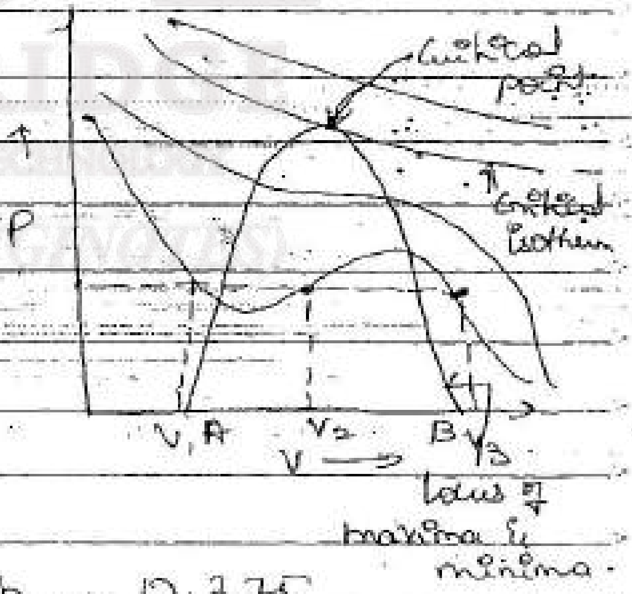
Now, $T_c = \frac{8a}{27Rb} = \frac{8 \times 27b^2 P_c}{27Rb} = \frac{8bP_c}{R}$

$$\therefore \boxed{b = \frac{RT_c}{8P_c}}$$

$a = 27b^2 P_c$

$= 27\left[\frac{RT_c}{8P_c}\right]^2 P_c$

$a = \frac{27}{64}\frac{R^2 T_c^2}{P_c}$



$\therefore Z = \frac{PV}{RT}$

$Z_c = \frac{P_c V_c}{RT_c}$

$Z_c = \frac{(a/27b^2) \times 3b}{R(8a/27Rb)} = \underline{0.375}$

$Pv^3 - (Pb + RT)v^2$

If we plot pressure against volume, we get a family of isotherms. The locus of maxima and minima points of the curves has been shown by thick line ACB. The apex C of this curve is called the critical point. The values of pressure, volume & temperature corresponding to the critical point are called the critical parameters for a gas & are denoted by P_c, V_c & T_c.

Law of corresponding States

Two gases are said to be in corresponding states, "If the gases have same values for any two of the three reduced variables, they also have the same value for the third variable"

We have,

$$\left[P_{r_1} + \frac{3}{V_{r_1}^2}\right](3V_{r_1} - 1) = 8T_{r_1}$$

&

$$\left[P_{r_2} + \frac{3}{V_{r_2}^2}\right](3V_{r_2} - 1) = 8T_{r_2}$$

If $P_{r_1} = P_{r_2}$ & $V_{r_1} = V_{r_2}$, then $T_{r_1} = T_{r_2}$

Reduced Properties - Law of corresponding States

The major disadvantage of any compressibility chart is that a separate chart is required for each gas. To make it convenient it is possible to construct a single compressibility chart that could be used for all gases.

The ratios of pressure, temperature and specific volume of a real gas to the corresponding critical values are called reduced properties.

Reduced property is defined as
"The ratio of the property of the substance at the given state to the value of the same property at critical state of the substance"

∴ Reduced property, temperature $T_r = \frac{T}{T_c}$

Reduced pressure $P_r = \frac{P}{P_c}$

Reduced volume $V_r = \frac{V}{V_c}$

"r" represents the reduced property, c denotes the critical state

Generalized compressibility chart

In this the compressibility factor Z is plotted as a function of reduced pressure and temperature. According to these charts, one component substance which are at same reduced pressure and temperature have the same compressibility factor.

It provides the deviation from ideal behavior and gives results with an accuracy of within 5 percent.

other real gas equations:

Beattie-Bridgeman Equation of State.

$$P = \frac{\bar{R}T}{\bar{v}^2}\left[1 - \frac{C}{\bar{v}T^3}\right](\bar{v}+B) - \frac{A}{\bar{v}^2}$$

where $A = A_0\left[1 - \frac{a}{\bar{v}}\right]$, $B = B_0\left[1 - \frac{b}{\bar{v}}\right]$

The constants A_0, a, B_0, b and c are given in tables.

Benedict-Webb-Rubin Equation of State

$$P = \frac{\bar{R}T}{\bar{v}} + \left[B_0\bar{R}T - A_0 - \frac{C_0}{T^2}\right]\frac{1}{\bar{v}^2} + \frac{b\bar{R}T - a}{\bar{v}^3}$$
$$+ \frac{a\alpha}{\bar{v}^6} + \frac{c}{\bar{v}^3T^2}\left[1 - \frac{\gamma}{\bar{v}^2}\right]e^{-\frac{\gamma}{\bar{v}^2}}$$

where constants a, A_0, b, B_0, c, C_0 & α γ are given in tables.

From Vander Waal's equation

$$P_c = \frac{RT_c}{V_c - b} - \frac{a}{V_c^2} \longrightarrow \text{(i)}$$

$$\frac{dP}{dV} = \frac{-RT_c}{(V_c - b)^2} + \frac{2a}{V_c^3} \longrightarrow \text{(ii)}$$

$$\frac{d^2P}{dV^2} = \frac{2RT_c}{(V_c-b)^3} - \frac{6a}{V_c^4} \longrightarrow \text{(iii)}$$

$\times^{ly}$ eqn (ii) by $\left[\frac{2}{(V-b)}\right]$

$$-\frac{2RT_c}{(V_c-b)^3} + \frac{4a}{V_c^3(V-b)} = 0 \longrightarrow \text{(iv)}$$

Adding eqns (iii) & (iv)

$$-\frac{6a}{V_c^4} + \frac{4a}{V^3(V_c-b)} \longrightarrow \text{(v)}$$

÷ eqn (v) by 2a & $\times^{ly}$ by V_c^3

$$-\frac{3}{V_c} + \frac{2}{(V_c-b)} \rightarrow \frac{2}{V_c-b} = \frac{3}{V_c}$$

$$\boxed{V_c = 3b}$$

from eqn (ii), we find value for Tc

$$RT_c = \frac{2a}{V_c^3}(V_c-b)^2$$

$$RT_c = \frac{2a \times (3b-b)^2}{3b^3} = \frac{2a \times 4b^2}{27b^3} = \frac{8a}{27b}$$

$$\boxed{\therefore T_c = \frac{8a}{27bR}}$$

Now substitute the values of Tc, Vc in eqn (i)

$$\boxed{\therefore P_c = \frac{a}{27b^2}}$$

ll^{ly} Substituting the values of Pc, Vc, Tc

$a = 27b^2 P_c$ $\qquad$ $b = \frac{RT_c}{8P_c}$

$= \frac{27}{64}\frac{R^2T_c^2}{P_c}$ $\qquad$ $= \frac{V_c}{3}$

$= 3P_c V_c^2$

Problems:

Date:

(Jan 2014)

1) Determine the pressure exerted by CO_2 in a container of 1.5 m^3 capacity when it contains 5 kg at 27°C. (i) Using ideal gas.
(ii) Using Vander waals equation.

$a = 364.3\ kPa\ (m^3/kg\ mol)^2$; $b = 0.0427\ m^3/kg\ mol$.

Number of moles, $n = \frac{m}{M} = \frac{5}{44} = 0.1136$.

molar specific volume $\bar{v} = \frac{V}{n} = \frac{1.5}{0.1136} = 13.2\ m^3/kg\ mol$

(i) Ideal gas equation

$$PV = mRT = \frac{m\bar{R}T}{M}$$

$$P = \frac{m\bar{R}T}{MV} = \frac{5 \times 8.314 \times 300}{44 \times 01.5} = \underline{188.95\ kPa}$$

(ii) Using Vander waal's equation:

$$\boxed{P + \frac{a}{\bar{v}^2}}\ [\bar{v} - b] = \bar{R}T$$

$$P = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}^2} = \frac{8.314 \times 300}{13.2 - 0.0427} - \frac{(364.3)}{(13.2)^2}$$

$P = 183.576\ kPa$

Q. Determine the pressure in a steel vessel having a volume of 15 litres and containing 3.4 kg nitrogen at 400°C by using (i) Ideal gas (ii) Vander waals equation. Also calculate compressibility factor by using Ideal gas equation

For N_2 $R = \frac{\bar{R}}{M} = \frac{8.314}{28} = 0.2969 \text{ kJ/kg K}$

$PV = mRT$

$P = \frac{3.4 \times 0.2969 \times 673}{0.015} = 45295.46 \text{ kPa}$

for N_2, $a = 136.4 \frac{\text{kNm}^4}{(\text{kg mol})^2}$, $b = 0.0386 \text{ m}^3/\text{kg mol}$.

$\bar{V} = \frac{V}{n}$; $n = \frac{m}{M} = \frac{3.4}{28} = 0.1214$

$\therefore \bar{V} = \frac{0.015}{0.1214} = 0.1235 \text{ m}^3/\text{kg mole}$

$$\left[P + \frac{a}{\bar{V}^2}\right][\bar{V} - b] = \bar{R}T$$

$$P = \frac{\bar{R}T}{\bar{V}-b} - \frac{a}{\bar{V}^2}$$

$$= \frac{8.314 \times 673}{(0.1235 - 0.0386)} - \frac{136.4}{(0.1235)^2}$$

$$\boxed{P = 56961.91 \text{ kPa}}$$

$PV = ZRT$

$v = \frac{V}{m} = \frac{0.015}{3.4} = 0.0044 \text{ m}^3/\text{kg}$

$$Z = \frac{Pv}{RT}$$

$$= \frac{56961.91 \times 0.0044}{0.2969 \times 673}$$

$$\boxed{Z = 1.25}$$

3) A vessel of $2.75 m^3$ capacity contains 5kg of oxygen at 300K. Determine the pressure exerted by oxygen using Vander waal's equation. Compare the result obtained if oxygen is treated as ideal gas. The constants appearing in the Vander waal's equation have the values:
$a = 139.35 \times 10^3 \ N m^4/(kg\ mol)^2$ and
$b = 0.0314 \ m^3/kg\ mol$

Volume occupied by 1kg mole of oxygen

$$\bar{V} = \frac{2.75 \times 32}{5} = 17.6 m^3$$

$$\left[P + \frac{a}{\bar{V}^2}\right][\bar{V} - b] = \bar{R}_0 T$$

(or) $$P = \frac{\bar{R}T}{\bar{V} - b} - \frac{a}{\bar{V}^2}$$

$$P = \frac{8.314 \times 300}{17.6 - 0.0314} - \frac{139.35 \times 10^3}{(17.6)^2}$$

$P = 1.43$ bar

Gas constant for oxygen,

$$R = \frac{\bar{R}}{M} = \frac{8.314}{32} = 259.8 \ J/kg K$$

Using the ideal gas equation, $PV = mRT$.

$$P = \frac{mRT}{V} = \frac{5 \times 259.8 \times 300}{2.75}$$

$P = 1.417 \times 10^5 \, N/m^2$

$P = 1.417$ bar.

4) Determine the specific volume of H_2 gas when its pressure is 60 bar and temperature is 100K.

a) by using compressibility chart &

b) by using Vander waal's equation.

for H_2

$T_c = -239.76°C$ $\quad a = 0.25105 \times 10^5 \, Nm^4/kg\,mole^2$.

$P_c = 12.92$ bar $\quad b = 0.0262 \, m^3/kg\,mole$

$T_c = -239.76°C$ $\quad P = 60$ bar

$= 33.24$ $\quad T = 100K$

$$T_R = \frac{T}{T_c} = \frac{100}{33.24} = \underline{3.008}$$

$$P_R = \frac{P}{P_c} = \frac{60}{12.92} = \underline{4.65}$$

from chart,

P_R & T_R

$Z = 1.035$.

$\therefore PV = ZRT$

$$PV = Z \frac{\bar{R}}{M} T$$

$$V = \frac{1.035 \times 8.314 \times 100}{60 \times 10^2 \times 2}$$

$\underline{\underline{V = 0.0717 \, m^3/kg}}$

~~$V = \frac{0.864 \times 8.314 \times 100}{[illegible]} = 0.044 \, m^3/kg$~~

from $\left(P + \frac{a}{\bar{v}^2}\right)(\bar{v} - b) = RT$

$$\left[60 \times 10^5 + \frac{0.25105 \times 10^5}{\bar{v}^2}\right]\left[\bar{v} - 0.02622\right] = 8.314 \times 100 \times 10^3$$

$\bar{v} = 0.14\,m^3/kg$ mole.

$$V = \bar{v} \times n = \bar{v} \times \frac{m}{M} = 0.14 \times \frac{1}{2}$$

$$\boxed{\therefore V = 0.07\,m^3/kg}$$

Processes	workdone	U_{2-1}	Q
constant Pressure Process	$W = P(V_2 - V_1)$ $W = mR(T_2 - T_1)$.	$mC_p(T_2 - T_1)$	$mC_p(T_2 - T_1)$. (or) $H_2 - H_1$
constant volume.	$W = 0$.	$mC_v(T_2 - T_1)$	ΔU.
Reversible hyperbolic (or) Isothermal-	$P_1V_1 \ln\left(\frac{V_2}{V_1}\right)$ $P_1V_1 \ln\left(\frac{P_1}{P_2}\right)$	0	$Q = W$.
Polytropic process.	$\frac{(P_1V_1 - P_2V_2)}{n-1}$ (or) $\frac{mR(T_1 - T_2)}{n-1}$	$mC_v(T_2 - T_1)$	$-mC_v \times \left[T_2 - T_1\right] \left[\frac{\gamma - n}{1-n}\right]$

Relationship among P, V and T in a reversible polytropic process.

$$PV^n = C.$$

WKT $PV = mRT$ (from ideal gas)

$$P = \frac{mRT}{V} \quad \text{(or)} \quad \left[\frac{mRT}{V}\right] . V^n = C.$$

$$\text{(or)} \quad TV^{n-1} = \frac{C}{mR}.$$

$$\therefore TV^{n-1} = C.$$

III^{ly} $PV = mRT$

$$V = \frac{mRT}{P}$$

$$P\left[\frac{mRT}{P}\right]^n = C \quad \text{Con.} \quad T^n p^{1-n} = \frac{C}{(mR)^n}$$

$$\therefore \; T^n p^{1-n} = \text{constant}$$

(i) $PV^n = \text{Const}$

$$P_1 V_1^n = P_2 V_2^n$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n$$

(ii) $TV^{n-1} = C$, $T_1 V_1^{n-1} = T_2 V_2^{n-1}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

(iii) $T^n p^{1-n} = C$

$$T_1^n p_1^{1-n} = T_2^n P_2^{1-n}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{(n-1)/n}$$